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THE MATHEMATICS TEACHER

Volume XXXIX Number 6

Edited by William David Reeve

Demonstrative Algebra*

By E. R. Stabler
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High School or college algebra, in comparison with high school geometry, is commonly recognized as a loosely organized subject. In algebra, the usual emphasis is on generalization of the concepts and rules of arithmetic, and on the use of a more powerful symbolism to solve numerical problems, but not on the systematic, logical development of the subject from a set of postulates and undefined terms. Standard terminology distinguishes between informal geometry of the junior high school and demonstrative geometry of the senior high school, but in the field of algebra we generally have only such designations as elementary, intermediate, advanced, or college algebra. These various courses certainly differ in the complexity of topics considered, and in the later courses somewhat more attention is probably paid to such fundamental postulates as commutative and associative laws, and more proofs of specific theorems may be given. But it can hardly be said that under usual conditions the logical structure of the subject, as a whole, is significantly much more advanced in the later than in the earlier courses. Somehow, we do not hear of courses entitled "demonstrative algebra."

* Presented at the fourth annual meeting of the Metropolitan New York section of the Mathematical Association of America at Brooklyn Polytechnic Institute, April 21, 1945.

Let us consider what this means from the standpoint of a typical student who has pursued his mathematical education throughout high school, and, say, through a full year course in differential and integral calculus in college. Such a student must look back upon his one year of high school plane demonstrative geometry, plus the additional half-year of solid geometry, as his only mathematical courses devoted primarily to proving theorems. The other courses are undoubtedly associated in his mind with such activities as learning new symbolism, problem solving, derivation of specific formulas, and ways and means of performing certain required operations. The student in question will recognize in a general way that both algebra and geometry form a necessary foundation for the study of trigonometry, analytic geometry, and calculus. He probably will be able to name certain theorems of geometry which are of fundamental importance in the latter subjects. But in all probability he will view the role of algebra in these subjects as not much more than that of supplying rules of the "do" and "do not" type, together with the symbolic medium by means of which trigonometric identities are proved, equations of curves are manipulated, and limits are evaluated. Algebra as a coherent logical theory about

numbers, their operations and relations, is likely to be a rather remote concept.

The question arises as to what can be done in high school or college algebra courses (not excluding combination mathematics courses) to promote increased comprehension of logical structure in algebra, and thus to prepare for a better appreciation of the algebraic foundation used in more advanced mathematical subjects. Of course, there may be a real difference of opinion among teachers concerning the extent to which such an attempt may be desirable. But it is believed that it may be worth while at least to consider some of the possibilities in this direction. Before proposing an answer to this question, I shall refer briefly to what others have said in the past, and are doing in the present, in connection with this problem.

First, I quote from a well-known monograph by Professor E. V. Huntington, which was published over thirty years ago. After mentioning the lack of logic in algebra teaching at that time, as compared with geometry, he called attention to the possibilities of algebra for demonstrative purposes in the following words:[1]

> . . . On account of the simpler nature of the concepts with which it deals, algebra is better suited than geometry to serve as an illustration of what is essentially involved in mathematical reasoning. In geometry, the very concreteness and familiarity of the subject matter is apt to obscure the logical structure of the science, while in algebra, the more abstract character of the theorems makes it easier to fix the attention on their formal logical relations.

It is possible to take issue with this comparison, but it seems unfortunate that there has not been some general experimentation with demonstrative algebra in the years since this was written.

Apparently there is a restricted amount of experimentation of this nature now under way. A few recent textbooks, especially in the mathematical appreciation category, and exceptionally in texts emphasizing standard subject matter, have included logical treatments of varying type and extent of elementary algebra, or the number system.

An article by Professor E. B. Mode of Boston University shows what one teacher has been doing by way of emphasis on one of the fundamental principles of algebra.[2] His methods are significant from the standpoint of many-sided interpretation of abstract algebraic postulates.

Finally, attention should be given to a stimulating article by Professor E. P. Northrop of the University of Chicago.[3] This was an address delivered at the meeting of the Mathematical Association of America in Chicago in November 1944. Professor Northrop outlined a course in mathematics given at the University of Chicago undergraduate College as part of the required College curriculum. In view of the policy of the College of admitting students after only two years of high school, it presupposes no more than one year each of high school algebra and geometry. The course outlined contains four interrelated parts, as follows: (1) logical structure, including an introduction to postulational methods, (2) geometry reviewed logically on the basis of a new set of postulates, (3) algebra, (4) coordinate geometry and trigonometry. The algebra part of the course, to quote the author, "centers around a relatively rigorous development of number systems which in turn leads to a relatively thorough investigation of variables and functions." As might be expected, the number system development mentioned is of a more postulational nature than the sophisticated type of constructive development familiar in graduate courses in function theory. From the description of the course,

[1] E. V. Huntington, "The Fundamental Propositions of Algebra." In *Monographs on Topics of Modern Mathematics*, J. W. A. Young, ed., New York, Longmans Green and Co., 1927 (copyright 1911).

[2] E. B. Mode, "The Commutative Law," *The Mathematics Teacher*, vol. 38 (1945), pp. 108–111.

[3] E. P. Northrop, "Mathematics in a Liberal Education," *American Mathematical Monthly*, vol. 52 (1945), pp. 132–137.

it appears to represent an unusually ambitious type of integrated program aimed to teach appreciation not only of demonstrative geometry and demonstrative algebra, but also in a broader sense of demonstrative mathematics. At the same time, the course aims to teach methods and techniques as well as appreciation.

However, in view of the general reluctance to follow the Chicago plan of starting college in "Grade 11," it does not seem likely that such a comprehensive and logical mathematics program will be able to dislodge the usual high school and college courses very rapidly. Our original question as to what can be done in connection with high school or college algebra to promote increased comprehension of logical structure will probably continue to have significance for some time to come.

As a sample of what might be done, it is proposed to exhibit here in outline form a simple logical unit of algebraic postulates and theorems for what may be called "systems of rational operations," together with a brief indication of further lines of development and possible coordinating material. It is not intended to go into specific pedagogical questions, such as when, where, or how the material might best be presented in the classroom. Presumably it would come at a relatively mature stage, and would be especially for the benefit of superior students. Some teachers might prefer to use material of this nature as a basis for mathematics club programs, instead of for formal class work. The outline of the unit follows.

Systems of Rational Operations

Postulates

Given a set K of numbers $a, b, c \cdots$ with operations "plus," "times"

0. For every pair of numbers a, b in K there is a uniquely determined number $a+b$.

0′. For every pair of numbers in K there is a uniquely determined number $a \cdot b$.

---

1. If a, b are numbers in K, the number $a+b$ also belongs to K.

1′. If a, b are numbers in K, the number $a \cdot b$ also belongs to K.

2. If a, b are in K, then
$$a+b=b+a$$

2′. If a, b are in K, then
$$ab=ba$$

3. If a, b, c are in K, then
$$(a+b)+c=a+(b+c)$$

3′. If a, b, c are in K, then
$$(ab)\cdot c=a\cdot(bc)$$

4. There exists in K a unique number 0 having the property that for every number a in K
$$a+0=a$$

4′. There exists in K a unique number 1 having the property that for every number a in K
$$a\cdot 1=a$$

5. For every number a in K, there exists in K a unique number $\bar{a}$ such that
$$a+\bar{a}=0$$

5′. For every number a in K, with the possible exception of 0, there exists in K a unique number $/a$ such that
$$a\cdot /a=1$$

6. If a, b, c are in K, then $a(b+c)=ab+ac$

Theorems

Th. 1. $\bar{0}=0$

Th. 1′. $/1=1$

Th. 2. $\bar{\bar{a}}=a$

Th. 2′. $//a=a$ provided $a\neq 0$

Th. 3. If a, b are any two numbers in K, there exists in K a unique solution of the equation $x+b=a$, namely
$$x=a+\bar{b}$$

Th. 3′. If a, b are two numbers in K, with $b\neq 0$, there exists in K a unique solution of the equation $x\cdot b=a$, namely
$$x=a\cdot /b$$

Def. If $x=c$ is the unique solution of the equation $x+b=a$ we write $c=a-b$

Def. If $x=c$ is the unique solution of the equation $x\cdot b=a$ $(b\neq 0)$ we write $c=a/b$

Th. 4. $a-b=a+\bar{b}$
$a-\bar{b}=a+b$

Cor. 1. $a-a=0$

Th. 4′. $a/b=a\cdot /b$ $(b\neq 0)$
$a//b=a\cdot b$

Cor. 1′. $a/a=1$

Cor. 2. $a-0=a$
Cor. 3. $0-a=\bar{a}$ etc.

h. 5. $b-a=\overline{(a-b)}$

Th. 6. $\bar{a}+\bar{b}=\overline{(a+b)}$

Th. 7. $a+(b-c)=(a+b)-c$ or $a+b-c$

Th. 8. $a-(b-c)=(a-b)+c$

Th. 9. $(a-b)+(c-d)=(a+c)-(b+d)$

Th. 10. $(a-b)-(c-d)=(a+d)-(b+c)$

Cor. 1. $a-b=c-d$ if and only if $a+d=b+c$

Cor. 2′. $a/1=a$
Cor. 3′. $1/a=/a$ etc.

Th. 5′. $b/a=/(a/b) \quad (a, b\neq 0)$

Th. 6′. $/a\cdot /b=/ab \quad (a, b\neq 0)$

Th. 7′. $a\cdot (b/c)=(a\cdot b)/c \quad (c\neq 0)$

Th. 8′. $a/(b/c)=(a/b)\cdot c \quad (b\neq 0, c\neq 0)$

Th. 9′. $(a/b)\cdot (c/d)=(a\cdot c)/(b\cdot d) \quad (b, d\neq 0)$

Th. 10′. $(a/b)/(c/d)=(ad)/(bc) \quad (b, c, d\neq 0)$

Cor. 1′. $a/b=c/d$ (where $b, d\neq 0$) if and only if $ad=bc$

Th. 11. $a(b-c)=ab-ac$

Th. 12. $a\cdot 0=0$

Th. 13. Division of any number different from 0 by 0 is impossible in K, and division of 0 by 0 is indeterminate.

Th. 14. If $ab=0$ where a, b are in K, then either $a=0$ or $b=0$, that is, there are no "divisors of zero" in K.

Th. 15. $a\bar{b}=\bar{a}b$ and $\bar{a}\bar{b}=ab$

Th. 16. $\bar{a}/b=a/\bar{b}=\overline{(a/b)}$ and $\bar{a}/\bar{b}=a/b$

Th. 17. $(a+b)/c=a/c+b/c$

Th. 18. $a/b+c/d=(ad+bc)/bd$

The particular set of postulates chosne will be recognized by those familiar with the terminology of modern abstract algebra as essentially postulates for a *field*, although stated in a concrete rather than abstract form. The postulates can be presented to students directly as assumed properties of the system of all rational numbers. However, the presupposed set K of numbers referred to in the postulates, need not, and preferably should not be labelled explicitly as the set of all rationals, for reasons of flexibility in interpreting and extending the postulates. An attempt has been made to unify the postulates and the subsequent development by using the parallel column form of statement as far as possible, thus exhibiting the partial duality existing between the operations of addition and multiplication.

On close examination it will be noted that the preliminary postulates 0 and 0′ cover the usual axioms of equality to the effect that if equals are added to, or multiplied by, equals the results are equal. The latter rules are equivalent to the statements that addition and multiplication yield unique results, and accordingly seem more closely connected with the nature of the operations themselves than with the nature of equality. If desired, the fundamental rules of equality, such as reflexivity, symmetry, and transitivity, may be listed as formal axioms.

The postulates numbered 1 to 5 and 1′ to 5′ may be referred to by the following familiar names with respect to each operation: closure, commutative and associative postulates, existence of unique identity, and existence of unique inverse. The latter postulates can be weakened, if desired, by deleting the word "unique." Postulate 6, the distributive postulate, has to straddle the two columns. The notation for inverses is made parallel by writing $\bar{a}$ ("bar a") for the opposite of a, that is, the inverse of a with respect to addition, and $/a$ ("slant a") for the reciprocal of a, or inverse of a with respect to multiplication. Students might be encouraged to use some such notation as this to help avoid preconceived notions concerning negative numbers and reciprocals, and also to avoid thinking of the opposite of a necessarily as a negative number, or the reciprocal of a necessarily as an or-

dinary fraction with numerator 1 (although frequently this may be desirable.)

The proposed development from the postulates may be divided into two parts: the first part, through Theorems 10 and 10′, consisting of those theorems which are consequences of the first five pairs of postulates, and the second part including those theorems which make use of Postulate 6, the distributive postulate.

The first group of theorems, like the postulates on which they are based, are listed in parallel columns, with slight qualifying clauses in the statements of certain theorems in the multiplication column. It is obviously sufficient to give the proofs for the left column alone, although students might be encouraged to work through both columns (with special note of the qualifications in the right hand column.) It will be noted that the theorems in the left hand column include, in effect, the usual rules for addition and subtraction of positive and negative numbers, as well as rules for handling parentheses. At the same time, the theorems in the right hand column include the standard rules for multiplication and division of fractions. This method of organization serves to emphasize close similarities which often go overlooked. Thus, referring to Theorems 7–7′ and 8–8′, the rules for removing or changing the position of parentheses enclosing the difference of two numbers and preceded by a plus or minus sign, are seen to correspond precisely to the rules for multiplying or dividing a number by the quotient of two other numbers. In certain cases, a theorem in one column is more important than the corresponding theorem in the other column. Thus Theorems 9 and 10 are listed mainly because of the importance of their duals. These theorems, and the corollary of Theorem 10, suggest that it may be useful to pair numbers by taking their differences as well as by taking their quotients, a procedure which is actually followed in introducing negatives in the constructive development of the number system.

The second group of theorems, beginning with Theorem 11, includes theorems on 0, multiplication and division essentially with positive and negative numbers, addition of fractions. Students should realize the logical reason for discontinuance of the parallel columns—namely, the fact that the dual of the distributive postulate (which would read $a+\overset{bc}{\cancel{ab}}=(a+b)(a+c)$) does not hold. One of the most important results for later mathematics is Theorem 13 stating flatly that division by zero is impossible, or indeterminate, in any system of numbers satisfying the postulates. Students who become thoroughly aware of this theorem and its logical background might be expected to be in a more favorable position to meet this bugbear at numerous points in the study of trigonometry, analytic geometry, and calculus. An instructive exercise would be to trace the logical ancestry of Theorem 13. It will be found to depend ultimately on all thirteen of the original postulates.

Also significant is the corollary concerning absence of divisors of zero.[4] For contrast, it might be helpful to present elementary examples of systems in which there *are* divisors of zero. Thus, let K be the system of integers modulo 4, or modulo 6, with $a+b$, $a\cdot b$, defined in the usual way. For the modulo 4 system we have $2\cdot 2=0$, and for the modulo 6 system we have $2\cdot 3=0$. Since a theorem of the development is contradicted, it will be recognized that not all of the postulates can be satisfied. Actually in each case only one postulate fails—the inverse postulate for multiplication.

Examples of systems in which certain of the other postulates fail, in addition to the obvious example of the system of natural numbers, might well be instructive. Thus, let the set K consist only of $0, 1, -1$, with $a+b$, $a\cdot b$, defined as usual. Then the closure postulate for addition fails, while all others are satisfied. Or again, let the set K refer to all positive integral

[4] A *divisor of zero* is defined as one of two non-zero numbers (or in abstract systems, elements) whose product equals zero.

exponents of a given positive number base. Then the only postulates which fail are those requiring the existence of 0 and inverses in the system. This fact in itself could be used as motivation later on for the desirability of extending the definitions of exponents to include positive, negative and fractional exponents. The extended system of exponents then satisfies all of the postulates.

Examples such as these seem important if students are to appreciate the true significance of the original postulates—as properties of the system of all rationals which may or may not be completely satisfied for other systems of numbers. Incidentally, an example of a finite system which does satisfy all the postulates can easily be given—merely take a system of integers modulo any prime number.

It will be noted that the development from the postulates could easily be continued to include theorems on such topics as special products, factoring, simultaneous linear equations, exponents. However, students should be led to see that the original postulates, by themselves, are inadequate for much further progress. A fairly satisfactory elementary development could be given by adding some postulates of order, or inequalities, thus permitting formal distinction between positive and negative numbers, and by adjoining two existence postulates, one providing for positive nth roots of positive numbers and the other insuring the existence of the number i.

Finally, for purposes of comparison with more advanced methods, students might be informed briefly of the constructive approach to the number system. This approach starts from assumed properties of the system of natural numbers, and then *defines* the systems of positive rationals and of all rationals, together with their operations, in such a way that it is possible to *prove* that the postulates for a field actually are satisfied for the system of all rationals. Then the system of all reals and finally the system of complex numbers are constructed by use of appropriate definitions, again permitting us to prove that these postulates, and various others, are satisfied. In other words, the constructive approach and the postulational approach are supplementary to each other in much the same way that necessary and sufficient conditions are supplementary. If we desire the postulates to hold in any abstract system of numbers, the theorems appear as necessary consequences. Conversely, when we construct in a concrete manner the real and complex number systems by use of definitions, for the most part corresponding closely to these consequences, then the properties specified in the postulates become proved as theorems.

To summarize, let us say that the inclusion of a logically developed unit for systems of rational operations, at some appropriate stage of the work in high school or college algebra might have certain valuable results. *First*, the plan would tend to counteract the impression of students that geometry is the only foundational subject in mathematics which can be logically developed from postulates. *Second*, the plan would unify and make more significant a number of important rules of elementary algebra, and would help make algebra a more meaningful foundation for such subjects as trigonometry, analytic geometry, and calculus. *Third*, it would give an especially prominent place to the important theorem that division by zero is impossible or indeterminate, a result which does not seem to register with students on the basis of the usual treatment. *Fourth*, it would provide opportunity for a brief introduction to postulational systems of varied types as they enter into modern abstract algebra and the foundations of mathematics. *Fifth*, it would call attention, by virtue of its very incompleteness, to the desirability of a complete construction of the real and complex number systems, as a logical sequel to the postulational development.

Proof in Arithmetic

By Dorothy Welch, *Campus School, Iowa State Teachers College, Cedar Falls, Iowa* and Herbert F. Spitzer, *The State University of Iowa, Iowa City, Iowa*

Proof as an aid in securing understanding has always been an essential part of the teaching of advanced courses in mathematics. Within the last twenty years teachers of arithmetic have begun to realize more fully the necessity of making sure that their pupils understand the processes with which they work. Many writers have suggested use of proof as one way of ensuring that the arithmetical processes children use are understood. Unfortunately the writings of authorities do not include either a definite statement of the nature of the proof to be used or a description of its operation in an instructional program. This paper is a report of a study which had its two major purposes (1) the definition of proof to be used in arithmetic and (2) a description of an instructional program that makes use of proof.

For clarifying the first purpose, that of of defining proof, the existing professional literature is not of great value. Most of what has been written is concerned with a reasoning type of proof, the relation of one idea to another, and places little dependence on the observations of the senses. That type of proof is used extensively in geometry, but is extremely difficult even for adults in the field of arithmetic.

Of the few specific procedures for obtaining evidence of proof in arithmetic found in the literature, checking is by far the most frequently mentioned. In checking, another process is usually used. For example, the correctness of an addition is determined by subtracting one of the addends from the sum, or the correctness of a multiplication is found by dividing the product by either the multiplier or the multiplicand. While the use of another process undoubtedly aids in the understanding of the process being studied, it does not provide proof of as thorough understanding as that provided in geometric proof. Furthermore, checking can easily become very mechanical and, therefore, contribute little to understanding.

Proof in the mathematical sense is seldom used in the arithmetical literature pertaining to grades one, two and three, but there is much mention of procedures which make for meaning and understanding. The suggested procedures for making concepts and processes meaningful make extensive use of actual objects, of substitutes for the objects such as fingers, sticks and the like, of pictures, and of representative drawings. All the devices just mentioned depend very much upon observations of the sense of sight. For example, in showing that his statement "There are five more school days until Thanksgiving" was correct, Jim said: "Here is a mark for Thursday, this one is for Friday and these three are for the days next week. That makes five." In this situation Jim used marks for days. Thus he was able to experience through the visual sense at least all the days simultaneously, and by use of marks he was able to count with minimum effort the total number. This use of counting, a process in which the child has confidence, is somewhat similar to use of theorems and axioms in geometric proof.

Proof as used in the arithmetic program of the University Elementary School,[1] the program studied for this report is of the type used in the preceding paragraph. In the lower grades there is much use of objects and representative drawings with actual manipulation of objects used to demonstrate understanding of processes. In these grades observation through sense of sight plays an important role. This use

[1] University Elementary School, University of Iowa, Iowa City, Iowa.

of the senses is not in conflict with the mathematical definition of proof since the objects and drawings are not measured but counted. The objects or drawings merely serve as a tool in helping the child to establish relationships between ideas. The following examples of proof illustrate the procedure as used.

Problem 1. Jack had 15 marbles. He found 3 more, and then gave 7 to George. How many marbles did Jack have left?

Proof A. 7 he gave away

11111 11111 1 $\boxed{1111 \quad 111}$

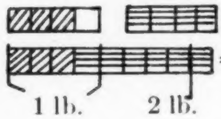
11 he had left 3 he found

15	18
+ 3	− 7
18	11

Proof B. Jack had 000000000000000
found 000 gave away
0000000
had 11 left 18−7=11

Problem 2. Susan went to the store and bought $\frac{3}{4}$ pounds of cheese and $1\frac{1}{2}$ pounds of meat. How much did Susan's two packages weigh?

Proof A. cheese meat meat



$=2\frac{1}{4}$ lb.

1 lb. 2 lb.

$\frac{3}{4}$
$1\frac{1}{2}$
—
$2\frac{1}{4}$ pounds

Proof B. cheese meat

Take this piece and put it in this place. That makes two whole pieces and $\frac{1}{4}$ more $=2\frac{1}{4}$ lb.

Problem 3. Jimmy has 25 marbles and his cousin Dick has 26 marbles. How many do they have altogether?

Proof A.

	tens	ones		
25	2	5	11111	11
26	2	6	111111	
—				
51	5	1		

11 is made out of 1 ten and 1 one.
Take the ten and put with the tens.
Put the 1 here.

Proof B.

tens	ones
xx	00000
2	5
xx	000000
2	6
xxxxx	~~0000000000~~0
5	1

In the examples above the diagrams show the facts presented in the problem, picture the operations performed, and show the answer to the question of the problem. In problem 1 the actual processes of addition and subtraction and the answer obtained are clearly indicated in each proof. The proofs for problem 2 indicate how fractions are added and the resulting answer. In the proof for problem 3, the numbers to be added are shown as tens and ones and the method of combining is indicated. Proof, then, is a diagramatic representation of the conditions stated and the operations essential in answering the question of a problem. Such a solution gives the child an opportunity to see the actual manipulation for which arithmetical computations are a substitute. Such proof also gives the child an opportunity to judge the accuracy of his computations, it results in confidence in computational procedures, and, of course, such proof provides a definite way of showing that the child understands what he is doing.

A description of an instructional program which makes extensive use of proof was the second purpose of the investigation being reported. Proof of the type described above is used in a variety of ways in the University Elementary School

program in arithmetic. In making this study, samples of proof were collected from grades one through six. These samples consisted of work of pupils on inventory tests given at the beginning of the year (a few of the items on the inventory test required the use of proof), samples of daily work selected by the room teachers, shorthand accounts of discussions of proof obtained during one semester, and tests prepared by the investigator and administered at the close of the first semester.

Samples of the type of proof obtained from the work of the children have already been supplied in the early part of this paper. Many of the proofs were not as complete as those shown but most proofs could be interpreted by the investigator. On a few occasions the classroom teacher was able to supply an explanation of diagrams that were not clear.

The shorthand accounts of work in the arithmetic classroom were made in order to have some evidence of how the various diagrams and drawings were interpreted by the pupils. Visits to the classrooms provided an opportunity to see the type of work done in each grade and to hear first hand the comments on, questions about, and explanations of the illustrative solutions. The shorthand accounts made of these class recitations or discussions were almost essential to understanding of the method of proof. Often the real worth of a child's proof was brought out by his explanation in answer to the question of another pupil in the class.

Through analysis of the samples of proof obtained from the children and observation of the arithmetic work in the university Elementary School five important uses of proof were determined. These uses are (1) in demonstrating understanding of facts and processes, (2) in showing the relationship between new processes and those the child has already mastered, (3) in showing that an answer to a number question is correct, (4) in helping a child to solve a problem which he is unable to solve by the approved computational procedures, (5) in demonstrating the need for more economical means of solving problems.

Of the various ways of using proof, that of demonstrating understanding is the most important and the one used most frequently. In first grade this may be showing of fingers or the making of marks to show what a number means; in second grade it may be the use of marks to show that $8+5=13$; in sixth grade it may be the drawing and rearranging of 8 one-thirds to show that $8\times\frac{1}{3}=2\frac{2}{3}$. This use of proof is by no means limited to problems but is often used in connection with examples. Even when a child is practicing to fix a fact such as $14-8=6$, he may be asked to show with marks that $14-8=6$.

Proof for showing the relationships between new facts and processes and the one the child already knows is used primarily when new things are being taught. For example, when the basic addition or multiplication facts are being developed proof will bring out the relationships between counting and adding and between counting, adding, and multiplying. Also, when proof is used in the introduction to the addition of two figure numbers, the relationship between the addition of tens and of ones is clearly shown. Proof when used in the manner just described is a stepping stone, a sort of foundation for the more mature process which the child is to learn to use in such situations.

The use of proof in showing that a number solution is correct is fairly obvious. The five samples of proof shown in a previous section of this article are good examples of this use. Other good examples are the use of a drawing to show that $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$, to show that $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, and the use of diagrams to show tens times tens always equal hundreds. The same procedures used in proving that answers are correct might be used in the initial solution to a problem. For example, by means of a well constructed drawing the product of $\frac{1}{2}\times\frac{1}{4}$ can be obtained by a child who does not know the process of multiplying fractions.

The use of proof to demonstrate a need for more economical means of solving problems is a relatively minor use of the procedure. It is not difficult to see how proof can be used for this purpose. A glance at the proofs and number solutions for problems 1 and 2 shown above will show that in economy of writing if in no way the number solution is superior.

In addition to uses of proof mentioned above the three following conclusions were reached. First, the samples taken from different work situations show a great variety in methods of showing proof. This is evident among pupils of the same grades as well as among those of different grades. Such variation indicates independent pupil activity, and allows for the individual differences that exist in any group of children of varying ages and interests.

A second conclusion reached as a result of studying the samples is that the proof used in the various grades gives evidence of some measure of understanding on the part of the children. They seem to know the meaning of the number system with which they work, they can explain what their diagrams mean, and hence what the problems themselves mean, and they gradually work out number solutions from their diagrams. Those who do not hit upon the accepted adult method for any particular type of problem will at least have had the advantage of individual work on the problem and a knowledge of the basis upon which the number solution used by adults is built. The children seem to know what they are doing and why they do it; they seem to see sense in what they do.

A third conclusion reached is that there is an increasing ability in the use of proof as the children have more experience with it. This can seen by comparing the types of diagrams used by the class as a whole at the beginning and end of the semester, and by comparing the samples of the work of separate children within classes. Improvement in the use of proof is shown also in the greater facility in the types of diagraming used. Such devices as grouping, shading, labeling, and the use of a key are gradually acquired as helpful aids to diagraming and understanding. The meaning of the number system and an understanding of the relations between the numbers seem to be acquired gradually as a part of the arithmetical background of knowledge of these children. In all cases, this method of learning arithmetical processes takes much time, but eventually the necessary concepts are formed and the meanings understood.

Proof shown in the semester's work of the children of the six grades of this school seems to indicate a good and growing knowledge of the necessary arithmetical concepts underlying the processes introduced at various levels. The children work individually, each progressing at his own rate, and each developing his own number solution until the method agreed upon as the most economical is finally reached. This method agreed upon is, of course, the best method in current use in texts and in life outside of school. These children seem to know what they are doing and why they do it, and to be able to work problems out for and by themselves. How much of this ability is due to the method of proof used is of course impossible to say. However, it is evident that the children can explain the problems with which they work, and it seems reasonable to suppose that at least a part of this ability could be traced to the methods described here.

Individual Subscribers

We are still having a great deal of trouble and extra expense because members of the National Council whose subscriptions expire with the May issue do not renew before the October issue is mailed to subscribers. This year we are mailing the October issue to such members and we hope they will renew promptly so as not to miss the November issue.—The Editor.

The Atomic Bomb, Its Evolution

By Emery E. Watson
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To insure a background in proper perspective for the evolution of the atomic bomb, five major historical events are enumerated, namely:

1. The first is "Deductive Reasoning," which originally expressed itself in terms of Euclidean geometry and more recently in terms of non-Euclidean geometry.

2. The second is the invention of "The place value and the zero," conventions by which we express, with equal ease, numbers indefinitely large or indefinitely small. Their universal use renders their importance second only to that of the alphabet.

3. The third event gave us the telescope. By its aid we see and photograph some sixty billion stars and nebulae, and explore the universe to a distance of five hundred million light years in every direction.

4. The fourth is the discovery of the "Law of Universal Gravitation" the forerunner of the idea of "Natural Law."

5. The fifth, the "Theory of Relativity" was verified by the British eclipse expedition in 1919. Its importance in both science and philosophy has been world wide.

6. The sixth major event is the evolution of the atomic bomb first used as an instrument of war in July 1945. During the years from 1890 to 1945 science and mathematics laid the foundation for the "atomic bomb" by the discovery of the X-ray in 1895, the Becquerel rays in 1896, and radium in 1898. The decade from 1890 to 1900 has been called the most fruitful in the history of science.

In 1904 Rutherford predicted the existence of, and in 1906 he established the nuclear theory of the structure of the atom. In 1909 while experimenting with sub-atomic, electrically charged particles from radium he found that it required about a million shots to obtain one direct hit on the atomic nucleus. Some three decades later, during the year 1932, occurred the discovery of the electrically neutral particle now known as the neutron. By the aid of the neutron, direct hits on the atomic nucleus could be obtained almost every shot. Soon thereafter, scientists began to speak of molecules composed of atoms, of atoms composed of electrons, protons, and neutrons. Later it was found that the attractive force between the protons in very large atoms was only slightly in excess of the repulsive force between protons due to like electric charges. Hence the splitting (fission) of the large atoms seemed imminent. The year 1938 was the turning point in the quest for atomic energy by indicating that uranium 235 held the key. The amount of energy which could thus be released from one pound of uranium 235 is the equivalent of the energy in 3,000,000 pounds of coal; 2,000,000 pounds of gasoline; or 20,000 pounds of T.N.T. This discovery of uranium fission in 1939 enabled scientists to obtain the first samples of U-235 in 1940. The amount was about a millionth of a gram. By the methods then used it took about 100 hours to obtain this small amount. A little later greatly improved methods led to the actual construction of the atomic bomb, and its initial use in warfare. The experiments in 1941 soon indicated that the "technique" needed in obtaining pure uranium was not that of high speed electrically charged particles obtained from radium, but well moderated, slow speed, electrically neutral particles such as the neutron which can be shot or sucked directly into the heart of the atom thus generating by fission, two or more lighter atoms. Furthermore, the speed of the neutron can be regulated by the use of a moderator such as carbon or graphite, which serves to slow down its speed with-

out capturing too many of the newly freed neutrons. This is essential because slow neutrons are more likely to split uranium atoms than are fast ones. These discoveries constituted a long step toward supplying the uranium 235 or the new man-made element No. 94 called plutonium, now largely used in the construction of the atomic bomb.

The Manhattan Project's Plant on the Columbia River at Hanford, Washington, is the world's largest atom-making factory. It is devoted entirely to the mass production of plutonium atoms. Each production unit consists of a large block of graphite in which is placed uranium-metal cylinders sealed in aluminum tubes to protect the uranium from corrosion by the water pumped through the "pile" for the purpose of cooling. Once the pile is set in operation, it runs itself. There are always enough stray neutrons or cosmic rays to start a chain reaction which will continue at an even rate for an indefinite time.

All atoms are made of the same three elements, namely a central nucleon composed of protons and neutrons, around which revolve the electrons as satellites, a sort of a miniature solar system in which the region occupied by the nucleon is to the entire space within the atom as one is to 100 billion. If one could expand an atom until its circumference enclosed 100 acres, its nucleus would not be larger than a base ball. The electron is about 1845 times the size of either the proton or the neutron but has only about 1 over 1845 of the mass of either the proton or the neutron. In fact the nucleon is very dense and contains about 99.97% of the total mass within the atom. The diameter of the nucleon is about 10^{-13} cm. and is surrounded by a nearly empty space 10^{-8} cm. in diameter. A neutron bullet fired at an atom, if the speed is too great, may pass through the atom just as a very short wave traverses the atom without interruption.

These statements indicate something of the almost empty space in which the electrons move and arrange themselves in concentric shells according to well established laws namely the centrifugal and centripital force. The electrons rotate on their axes and revolve around the nucleon. Each part of the atom as well as the molecule is in constant motion.

The diagrams which follow show the schematic plan for the arrangement of the electrons within an atom, number 4 being the uranium atom. The inner shell contains 2 electrons: the fifth shell 18 electrons: the sixth shell 32 electrons and the seventh shell, the outer shell of the uranium atom, 6 electrons. The most stable condition of the atom exists when the outer shell has its full quota of electrons. Hence the very large atoms are unstable. Each electron carries a negative electric charge, each proton carries an equally large positive electric charge, and the electric charge on the neutron is zero. The chemical properties of the atom are largely determined by the number of electrons in the outer shell, the optical properties by the layer of the electrons further toward the nucleon, the x-rays and even shorter rays come from the disturbances of the electrons nearer to the nucleon.

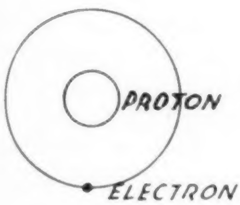
When referring to the molecule as being large we mean comparatively speaking. Its actual size is such that it cannot be seen even with the highest powered microscope. If a drop of water is magnified to the size of the earth, the molecules of which it is composed might resemble in size, a cluster of oranges.

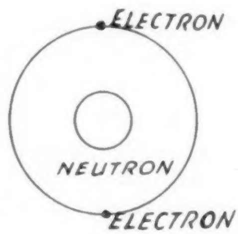
To verify this statement, it is possible to make an oil film one-fifty millionth of an inch in thickness. Consequently the molecule must have a diameter less than a wave length of light.

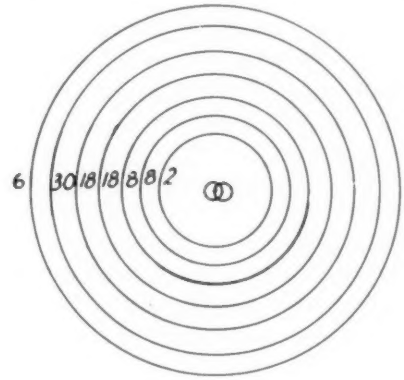
The simplest atom is the hydrogen atom. It is composed of one electron and one proton. The helium atom is composed of two electrons, two protons and two neutrons. These light atoms of hydrogen and helium are followed in order by atoms having three protons, four protons, and so on to the very heavy uranium atom, which

THE ATOM AND ITS PARTS

No. 1 In nucleus	Proton, mass = 1, electric charge = 1, Neutron, mass = 1, electric charge = 0.	
In outer orbit	Electrons, mass = 1/1845, electric charge = 1.	
Simplest atom No. 2	Helium atom No. 3	Uranium, Nature's heaviest atom. No. 4







Hydrogen atom.

Atomic number = 1

Atomic wt. = 1

Proton = 2, Mass = 1.
Charge = 2 +.
Neutrons = 2, mass = 2.
Charge = 0.
Electrons = 2, mass small.
Charge = −1, atomic wt. = 4.

Schematic distribution of electrons,—Lewis and Langmuer.

In one lb. of helium, the nuclear energy = the amount required to light a 100-watt bulb for 13,000,000 years.

is at the upper end of the periodic scale and has 92 electrons, 92 protons, and 146 neutrons, 330 particles in all. Hence atoms, such as radium, No. 88 and uranium No. 92 are all, comparatively speaking, large and heavy, the latter atom having 238 times the mass of the light hydrogen atom. As a result the attractive force which binds these particles together is not sufficient in every case, to overcome the repulsive force between the like electric charges on the protons. Hence these unstable atoms may throw off particles. This is expressed by saying they are radioactive. Radioactive atoms may disintegrate and in some cases do so very rapidly. In fact atoms heavier than uranium are never found in nature. The question may arise, "Why are there only 92 elements in matter?" The answer is, "Any nucleon" heavier than that of uranium No. 92 breaks down by radioactivity and destroys itself.

The names of three of these heavy radioactive atoms, namely Nos. 92, 93, and 94 are readily fixed in mind by remembering that they were named for the outer planets, namely, Uranus, Neptune, and Pluto. The atomic weight of uranium is 92, of neptunium 93, and plutonium 94. The latter element is a new chemical element never seen in nature.

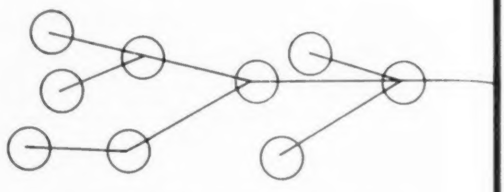
Atoms as well as electrons are in continuous motion. When two atoms approach each other, the outer cloak of electrons of one repel the outer cloak of electrons of the other, thus preventing the atoms of one from entering the domain of the other.

The electron was once thought to be an independent entity created from the energy of space or composed of hydrogen. Recently it has been found that the electrons, both positive and negative, may be formed from the nucleon. The neutron may change into an electron and a proton.

One result of this change is the possible formation of a new element called "neutrino" (the little neutron). The neutrino is thought to have a very short wave length. Like the neutron it has no electric charge, no ponderable mass, and can penetrate solid iron many thousand of miles in thickness. That is it can pass through, so-called matter with little or no interference. It is now known to be possible to bring about a condition in which both negatrons and positrons are transformed into the energy of space.

It is well known that an electric current can decompose water molecules into atomic elements, namely, one atom of oxygen to two atoms of hydrogen, one element at one pole and one at the other. If the gases are mixed, an electric spark or a lighted match will explode them forming water. In a way, this action suggests somewhat remotely, the action of the atomic bomb. The element used in splitting the atom and thus releasing an almost untold amount of atomic energy is the neutron. It may be obtained by embedding a small amount of radium mixed with berillium in a block of paraffin. The neutron tends to split the atom and release an immense amount of energy. The energy released is equal to 11,400,000 kilowatt hours for each pound of U-235 split.

By 1929 great machines were installed for splitting the atom. These machines called cyclotrons may be described as having a large circular chamber placed between the poles of two powerful electro magnets. Inside the chamber are two semi-circular electrodes called "dees." These "dees" are connected to a source of high frequency alternating current, thus creating an electric field which accelerates the particles and causing them to move in spirals. On reaching the outer rim of the chamber, they emerge as fast moving particles ready for atom smashing. These machines play an important part in the development of the atomic bomb.



Energy Released by Fission

Plutonium, Element 94.

By June 1940 it was fully realized that there were only two immediate sources of atomic energy, uranium No. 92, atomic weight 235 and plutonium No. 94 and atomic weight 240. That the later would yield atomic energy equal to that of uranium 235 was known as early as March 1941. By December 1942 it was demonstrated at the University of Chicago campus that plutonium could be produced in large quantities by means of a special lattice work arrangement of uranium and graphite. That plutonium can be created out of ordinary uranium 238 is of utmost significance.

The uranium atom, weight 238, when struck by a neutron does not split, but results in "neutron capture," which causes the nucleus of the atom to become unstable. The neucleon now emits an electron, a beta particle, and the resulting neutron is changed into a proton. The result is a new chemical element, No. 93, called neptunium. It has 93 electrons, 93 protons and 146 neutrons.

But the neptunium atom is also unstable, and in time a negative electron, a beta particle, is emitted by the nucleon. This leaves the neutron with only the positive charge that has served to neutralize it. It now changes to a proton and the resulting atom is composed of 94 electrons, 94 protons and 145 neutrons. This new element, called plutonium, can now be produced in relatively enormous quantities. Once separated, the elements for the formation of these bombs may be assembled in sub-critical masses and exploded later. In the final reaction the explosion is violent, almost instantaneous, and the detonation is great.

The Improvement of High School Mathematics Courses as Recommended by the Commission on Post-War Plans*

By James H. Zant
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The Commission on Post-War Plans conceives the problem of improving mathematical instruction as one which should be thought of in its entirety, that is, from grades one through fourteen and not broken into fragments as has been done so often in the past. It is our opinion that the program throughout all of these grades is in need of a thoroughgoing reorganization and the Second Report of the Commission makes a number of specific recommendations which it hopes will be of aid in such a reorganization.

The first report of the Commission, which was published in the May 1944 issue of The Mathematics Teacher, set out a list of tentative statements designed to focus the attention of those interested in the teaching of mathematics on pertinent issues in mathematical education. The second report, published in the May 1945 issue of The Mathematics Teacher, presents a series of constructive theses. These are merely statements of beliefs or principles which the members of the Commission as a whole are prepared to defend. They are the result of its sober judgement and careful thinking based on the best evidence available. These propositions are not to be regarded in any sense as final or perfect, nor are they offered in a dogmatic spirit. They do, however, reflect the sincere convictions of the group as a whole. It is hoped that these tentative guides will be widely discussed and suggestions as to changes will be welcomed at all times by the Commission. The Commission is seeking through the cooperative thinking of teachers of mathematics in the entire school system to arrive at a set of principles (a blue print) for building a stronger program in mathematical education. Quoting from the Commission's report, "The mental climate with respect to mathematics is at the moment very favorable. Now is the time to put our house in order by making the improvements which have been proposed for too long. In the post-war period we should be ready to provide the very finest mathematics program for every type of youngster in our schools. It is to that end that the Commission submits the theses for careful study."[1]

It is obviously not possible for me to discuss in this short paper all of the points of the Second Report of the Commission. Hence at the suggestion of our chairman I have stated the title of these remarks as "The Improvement of High School Mathematics Courses as Recommended by the Commission on Post-War Plans." As a keynote to the discussion I will quote again from the Commission's report and say that we as secondary school people must realize that "The high school needs to come to grips with its dual responsibility, (1) to provide sound mathematical training for our future leaders of science, mathematics and other learned fields, and (2) to insure mathematical competence for the ordinary affairs of life to the extent that this can be done for all citizens as a part of a general education appropriate for the major fraction of the high school population."[2]

The first thesis stated by the Commission deals with mathematical competency.

* Read on February 15, 1946 before the Oklahoma Section of the National Council of Teachers of Mathematics at the Biltmore Hotel, Oklahoma City.

[1] "The Second Report of the Commission on Post-War Plans." The Mathematics Teacher, Vol. 38, No. 5, pp. 195–196 (May 1945).

[2] *Ibid.*, p. 195.

It reads as follows: **Thesis 1.** *The school shall guarantee functional competence in mathematics to all who can possibly achieve it.* While it is obvious that the task of teaching the fundamentals of mathematics must be begun in the elementary grades, the task of seeing that pupils have functional competence before leaving school falls peculiarly upon the high school, since the pupils are there last. Hence, items which were not taught in the lower grades or which have been forgotten must be retaught during the high school years.

The Commission made no attempt to give an explicit definition of "functional competence." Progressive mathematics teachers have known for many years that pupils were leaving the schools at graduation time with definite shortages in their mathematical knowledge. This thesis states that the Commission believes that it is the responsibility of the whole school to do something about it.

However, a fairly definite basis for functional competency in mathematics is suggested. It is clear that modern technology has stepped up the minimum requirement in mathematics for effective citizenship and that it is already a big step higher than the fundamental operations of arithmetic. For example, life in the armed forces for a boy who was merely a good computor was decidedly limited. Even a small amount of training in high school mathematics is a valuable asset to a young person either in the armed forces or in industry.

Hence the Commission suggests a Check List of twenty-eight items which was based on reports of committees appointed to study essential mathematics a boy needs for the Army.[3] Until there is time for further investigation it seems sensible to assume that the mathematics essential for army needs will also serve, with only slight modifications, for the mathematics desirable for all our citizens. The Check List mentioned above appears in the Second Report and includes such items as, the fundamental operations with whole numbers, fractions, decimals and percents, an understanding of ratio, ability to use tables, simple statistics, understanding and use of geometric ideas like angles, polygons, solids, etc., measurement and measurement devices, the meaning and use of formulas, signed numbers, equations, similar triangles, the meaning and use of the mathematics of the home, community and business, and the like.

As was suggested earlier, this thesis is one of the main goals for every teacher of mathematics from Grade 1 to Grade 14. Much of this will be accomplished in the earlier grades and perhaps a course in general mathematics in the high school should be expected to complete the task. However, it is well known that many high school graduates are not acquainted with many of these items. The traditional sequential courses in high school mathematics, that is, algebra, geometry and trigonometry, as now taught do not insure these skills. Hence it may be necessary to reteach these things; it will certainly be necessary to see that they are known before the student leaves the secondary school. The time has passed for "passing the buck" to a teacher of some lower grade and excusing ourselves by saying that the pupil should have learned to add or solve simple equations before he got to this particular class. It is everybody's responsibility.

The other part of the dual function of the high school, that is, "to provide sound mathematical training for our future leaders of science, mathematics, technology and other learned fields" is also extremely important, though it deals with a much smaller group of individuals. The manpower and selective service policies of the government during the last four years

[3] See the report of the Committee on "Preinduction Courses in Mathematics," THE MATHEMATICS TEACHER, March 1943, pp. 114–124 and the report of the Committee on "The Essential Mathematics for Minimum Army Needs," THE MATHEMATICS TEACHER, October 1943, pp. 243–282.

have resulted in a drastic depletion of the scientific resources of the nation. Not only have well trained young scientists and technologists been used up in combat divisions, but the sources of such trained men have almost dried up. It has been estimated that by 1949, the first year which we can expect a regular sized graduating class, the country will be behind on trained engineers alone by 55,000. The total deficit of scientific and technological personnel "may, according to the Bush report, run as high as 150,000 at the bachelor's level and not less than 17,000 at the doctorate level."[4]

At the same time scientific discoveries and improvements in industry and manufacturing have created demands for more scientifically and technically trained men than we have ever needed before. Some of this need has been filled by women entering this field in increasingly large numbers, but with the return of normal living and normal demands for manufactured goods and service it will be hard to meet the new demands that will be made.

In addition to the reasons stated above the training of scientific personnel may be further hampered by the national policies toward universal military training. Whatever disposition is finally made of this problem by the Congress, it is important that provisions should be made to insure proper training for those young men who are capable of doing outstanding work in mathematics, science and technology. The high schools' part in this program will be to see that the students of more than average ability are given a thorough training in mathematics and science; a much more thorough training than most of them have received in the past. Ways of providing for this group will become apparent as this discussion continues.

While it is not desirable that any group of able-bodied young Americans should receive exemption from a universal military training program, it seems reasonable to recommend that those who have the ability to become outstanding in mathematics, science and technology should be allowed to continue their studies in both secondary school and college until they have completed enough work to enable them to enter some special service branch of training in the army or navy. This would mean that these young men's military training would be deferred until they have prepared themselves to enter on special training courses set up and needed by the armed forces. In this way their scientific training would not be interrupted at all, since their work in these special branches would be a definite addition toward their chosen field and there would not be the danger of them being shunted into some routine job before they had gone to school enough to realize their interests and capabilities.

Hence the Commission set up **Thesis 12.** *The large high school (with more than 200 pupils) should provide in grade 9 a double track in mathematics, algebra for some and general mathematics for the rest.* The general mathematics course is recommended so that the larger group who are not interested or capable of pursuing a scientific course may have this opportunity to learn the items discussed above under functional competency and also the mathematics which will enable them to render semi-skilled service in industry, the larger business concerns, small private shops and businesses, and the like. The organization of these courses in general mathematics also has the function of eliminating the group of students who lack interest and ability from the courses in sequential mathematics.

It is most important that the course in general mathematics shall be administered and taught in a way that will make it respectable and desirable. The attitude of the teacher is often a determining factor, and far too often the teacher is, by training disposed to propagandize for algebra. The principal and advisers often have a

[4] See *A.A.A.S. Bulletin*, No. 1, Vol. 5, p. 3, January 1946.

tendency to belittle the value of general mathematics and think of it as a place to put those students who are not quite good enough to be in the algebra class. Implying that the failure to succeed in the class in algebra is a reason for a transfer to the general mathematics class gives algebra an unwarranted halo and creates a stigma of disrespect for general mathematics which would not be the case if it were properly organized and taught.

"It should be made clear to the pupils that the two parallel courses of the ninth grade are both tremendously worthwhile, but they do have very different goals and experiences for pupils with different interests and needs. General mathematics is a more flexible course than algebra; it can more easily be adapted to backgrounds and levels of ability. The material can and should be offered in such a way as to challenge the pupil to his best effort. Pupils should be told that general mathematics is *organized* differently, that it offers a greater *variety* of topics and that it is related *more directly* with immediate application. They may well be told that good work in general mathematics demands as much time and exertion as algebra. It is a fatal error to imply that in general mathematics anything will do. Shop teachers of the right kind certainly expect accuracy in computation and measurement beyond anything required in the ordinary academic class. General mathematics may seem an easier course than algebra, but that fact alone will not stigmatize the course in the opinion of its pupils."[5]

Nor should classification be based on ability alone. It should be based on a difference in goals. If a pupil expects to continue the study of science and technology, he must know algebra; that is one of the prerequisites. However, some of the best pupils in the ninth grade may have no interest on this line and should therefore be in the course in general mathematics. It gives them an opportunity to obtain a mastery of the essentials as outlined in the Check List and will often serve to remove a feeling of insecurity from many who have never had the satisfaction of achievement in mathematics.

The selection of a pupil for the algebra class should be based on interest and the ability and desire to do work of a high order of excellence. Hence, unsatisfactory work in algebra should be tolerated for only a brief period—probably less than a semester. A poor knowledge or a merely mechanical knowledge of algebra at this stage is often worse than useless if the pupil tries to go on. However, it is again important to be sure that the pupil is not prejudiced against general mathematics but made to see that it is necessary for him to shift his goals and choose a course which will better serve his new purpose. It may well happen that at the end of a year or more of general mathematics a pupil may be encouraged to study algebra, provided there is convincing evidence of his ability and interest. "The goal of a strong mathematics department should be to have every pupil in the appropriate course with no dissatisfied customers."[6]

The Commission had much to say about the mathematics of grades 10 to 12. While there have been many attempts to correlate the so-called sequential courses in mathematics, that is, algebra, plane and solid geometry and trigonometry, in general they are taught separately. They have probably changed less than any subject in the curriculum but there is great opportunity for improvement.

Several theses of the Commission make definite suggestions along these lines.

Thesis 14. *The sequential courses should be reserved for those pupils who, having the requisite ability, desire or need such work.* Pupils who need this sort of mathematics must be taught many phases of mathematics that are more detailed and abstract than is necessary or desirable for the majority. Hence these courses must not be emasculated to fit the needs of those of

[5] Second Report, *op. cit.*, pp. 205–206.

[6] *Ibid.*, p. 207.

low ability and weak purpose.

Thesis 16. *The main objective of the sequential courses should be to develop mathematical power.* A pupil who must use mathematics in later study and in practical work must understand the underlying principles and concepts of its development. He must further understand the application of these principles to new and varied situations. Hence, the sequential courses must not be taught as mere drill and manipulations but as basis understandings, drill and applications.

However, as stated in Thesis 15, *teachers of traditional sequential courses must emphasize functional competence in mathematics.* It is important to stress again that it is a definite part of the high school's job to see that each student has acquired a sound knowledge of fundamental useful mathematics before he leaves the high school. Many students who have a thorough knowledge of the sequential mathematics do not know the more practical things such as consumer mathematics, simple statistics, the ability to use tables and the like. Hence, we must see that students who are interested in the sequential courses in mathematics are also required to make themselves competent in that part of mathematics which is needed by everyone.

As stated by the Commission, most of these topics can be fitted into the regular sequential work and become an integral part of it. For example, by applying the principles of algebra and geometry many numerical calculations can be introduced and good practice can be obtained in the use of whole numbers, fractions and decimals. Too often the algebra and geometry teachers take no responsibility in seeing that the student is able to perform such operations accurately. Topics not clearly understood should be retaught and adequate practice provided for those students who need it.

Thesis 20 deals with the vital problem of the small high school as related to the possibility of providing a rich offering in mathematics. It is stated thus, **Thesis 20.** *The small high school can and should provide a better program in mathematics.*

Since more than two-thirds of all high schools have less than 200 students and probably less than 8 teachers, it poses one of the vital problems in the mathematical education. There is little professional literature dealing with this problem. Thus "a considerable fraction of the population of our high schools has been overlooked altogether in formulating programs for the teaching of mathematics."

Common practice in our small high schools varies between two extremes. Many schools, probably a large majority, offer a year of a formal algebra and a year of geometry. The apparent aim of such a program is to prepare all students for college even though the school may have sent relatively few of its graduates to college in its entire history. At the other extreme the school may offer a year of commercial arithmetic and a year of agricultural or general mathematics with little emphasis on basic concepts and fundamental principles. In such schools a gifted pupil may graduate with a wholly inadequate knowledge of the sequential mathematics he will need if he wishes to continue his study of science, mathematics or technology in college.

The Commission has four definite suggestions to make for such small high schools.

1. Teachers of rural schools have always taught from six to eight grades in one room with several classes going simultaneously. It seems reasonable to suppose that the high school teacher can teach two. Hence it is proposed that the small high school *"offer two courses simultaneously within the same class period."* That is, instead of general mathematics or algebra, teach both courses at the same time.

2. Since a great variety of correspondence courses are now provided by both the state colleges and by private commercial organizations, it is proposed that the small high school make it possible for interested students to take such courses in mathematics.

During the war large numbers of men, well over a million, took correspondence courses under conditions which were often very difficult. Students in high school would have a much better chance to succeed in such courses, especially if the mathematics teacher is given the responsibility of supervising the students' work. Since classes in these high schools are small anyway, the teacher will be well able to to assume this responsibility.

3. A third suggestion is one which has been practiced in our Oklahoma schools for many years. It is to "*increase the number of courses offered by cycling.*" This, as many of you know by actual experience, makes it possible to have a much wider offering of courses during the ninth, tenth, eleventh and twelfth years than would otherwise be possible. In this way it is possible for the student who stays in the same community during most of the high school period to get an adequate number of courses in mathematics.

4. The final suggestion is one which is hard to comply with under present conditions of low salaries and the inadequate supply of teachers. The Commission suggests "that the superintendent and board of education employ at least one teacher with a minor or major in mathematics in order that the extended offering may be properly taught." It is obvious to those of us who deal with the graduates of all of the high schools of the state that no program of improvement in the teaching of mathematics will have much success unless the teachers have an adequate mathematical training themselves. Another thing which the Commission did not mention but which is implied here is the effect of the attitude of the school administrators toward the mathematics that is taught in their schools. No teacher can do a very good job when the high school principal and superintendent have no interest in or respect for the mathematics which is being taught in the classroom. On the other hand, it is easy for college teachers of mathematics to recognize students who have studied their mathematics in schools where the administration has a healthy sympathetic attitude toward mathematics and its value to their graduates.

Before concluding I would like to speak briefly of a new task which the Commission has set for itself during the next two years.

"The Board of Directors of the National Council of Teachers or Mathematics are agreed that the next job of the Commission is to create, or cause to be written, a relatively small but effective pamphlet on mathematics to be used in the guidance of the junior and senior high school pupil. Its purpose will be to make clear what mathematics has to offer. The idea is that it be directed to the student but widely distributed to mathematics teachers, who in turn may transmit the pamphlet or at least the ideas, to home room teachers and other persons with responsibilities for guidance."[7]

The Commission is now at work on some ten or twelve vocational "areas" or "categories" which will be separately treated and discussed. These areas are tentative and include Aviation, Engineering, Mechanical Trades, Agriculture, Scientists and Mathematicians, Civil Service, Business and Accounting, Actuarial Work, Statistical Work, and Nursing. Preliminary drafts of discussions of each of these topics have been prepared and were criticized at the meetings of the Commission in Cleveland on February 23 and 24, 1946.

The Commission considered two aspects of each of the above named topics. 1) The typical mathematical needs or demands made upon persons doing this sort of work and 2) the nature and amount of mathematics in high school and junior college, in terms of courses and or/topics, which a student should pursue if he is planning to enter the particular profession.

This is something which should have

[7] James H. Zant, "The Next Step in Planning for Post-War Mathematics," THE MATHEMATICS TEACHER, October 1945, p. 276.

been done long ago. We have all known that mathematics is needed in varying degrees in many types of work. In some cases, like engineering and actuarial work, the recommended courses have been carefully selected and definitely required. In many others the requirements have been hazy and uncertain. Very few mathematics teachers have any systematic knowledge of just how mathematics is used in business and industry, or what mathematics is needed by the student who wishes to prepare himself for specific jobs. Another thing which is badly needed is specific knowledge of companies and offices which can and will use a well-trained mathematics student when he is ready to go to work.

The Commission hopes to be able to assemble data which will answer these questions and others which interested students will want to ask. It has been difficult in the past to interest students in the study of mathematics because the job possibilities which teachers have known about have been so meager. The common answer has been that a person had to teach if he expected to make his living by the use of mathematics. This is not the case. There is a large variety of jobs available for graduates who major in mathematics. This is especially true if the student has received a certain amount of specialized training in mathematics applied to specific fields needed by business, industry and government service. It is being recognized that workers trained as engineers, physicists, accountants, etc., are not wholly qualified to solve the increasingly difficult mathematical problems which occur in industry and business. The teachers of mathematics in high school and college should have definite knowledge about these things. The students who are interested in the study of mathematics and who are capable of succeeding in the more advanced courses have a right to expect their teachers to have this knowledge in a definite and concise form so that they will know just what is expected of them if they go into specific lines of work. The Commission's task is to assemble this information for students at the high school level. It is our desire to make this data thoroughly reliable and have it answer questions which arise constantly for students and teachers in the secondary school. To do this we will need the help of the teachers in these schools at well as a fresh practical approach to the problem of guidance from the standpoint of the pupil and his future job.

This discussion of the work of the Commission on Post-War Plans is of necessity fragmentary. The Second Report dealt with several other topics. The future plans include, in addition to the creation of the guidance pamphlet, several follow-up projects on the Second Report. For example, several members of the Commission are working on a test on *Functional Competence in Mathematics* which we hope to standardize and make available to all mathematics teachers. Certain Hollywood studios and others have been interested in the production of movies and film strips for use in the teaching of mathematics. The members of the Commission are eager to receive suggestions at all times. We will also be glad to meet with groups of teachers large and small to discuss any of the problems raised by the reports of the Commission. The membership of the Commission represents all areas of the United States and we hope to make our work as widely representative.

The Role of Quantitative Thinking in Education

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With the physical violence of war at an end, it is easy for us in such a pleasant land as ours to live ostrich-wise in terms of immediate or prospective comforts. Warnings about atomic bombs are likely to have less effect than we imagine upon us, because horror is far away enough to be unreal, and we can always hope that somehow things will be all right.

Too, the growth of fine character is a slow achievement. It belongs to the deeper processes of life, akin to the growth of a tree in time, yet more subtle and more exposed to the fury of elements which can warp and shatter and destroy.

So also are the social achievements of fine character. The building of a sound home is a life-time task and requires persistance, energy and courage, to say nothing of stable vision.

Amid the insistent clamor of world need it is as though there was no possible way forward other than through quiet processes of growth, personal and communal, which seemingly cannot be hurried, which require a life-time at least for fulfillment, and which curiously enough are not for sale across store counters.

This then, it seems to me, is a first requisite for sound teaching, namely, a sound philosophy which is grounded in life realities.

Whatever special aspects of such a philosophy need emphasis in terms of the teaching of a particular subject, and there are such, these must never be allowed to obscure the more fundamental issues. Rather the teaching of the particular subject must be a vehicle of life-teaching, serving to re-inforce or to underline the particular aspect, but never to obliterate the basic purposes of living.

For too many people the question "what is life all about?" will be answered in the light of experience, with cynicism, which is always akin to Giant Despair.

Suppose, however, we begin abstractly with the world of geometry, the one dimension of the little child who loves to push objects back and forth along the single track of the window ledge. Soon we have two dimensions, length and breadth, not merely of the school blackboard, where alas still too often we leave our geometry, but the length and breadth of our wide prairie land, the expanse of the sea. . . . Now we have three dimensions, a world in which we can build bridges and fling up vast earthworks, in which the depths of space beckon our imaginings to the great infinitudes of the visible world. There is a grandeur and endless fascination about this spatial world, where we frail creatures contemplate its wonder, all the while confined to the two dimensional skin of a tiny whirling sphere!

But not for long are we content with space, for the fourth dimension of duration carries us out of the present, backwards to the spectacle of human history and all its antecedents which have baffled our best imaginings; and of the future—what shall we say?

With our four dimensions of space and time we have a stage nearly set. Despite the vast emptiness of space-time we all realize the presence of matter and of energy. So important, so fundamental are they, that there are those who see the whole scene in their terms. We are creatures, woven out of this four-fold fabric of space, time, matter and energy, And so we are.

Bertrand Russell once remarked that so much of our life is spent in the sheer displacement of matter. We carry parcels, books, dishes to and from the sink; we create vast transportation schemes to move the earth's materials including ourselves.

How much of our life is contained within the magic (or are they tragic?) words: space, time, matter and energy! We can see in the vast stores of matter and energy, employment and hence sustenance for all the races of mankind—there need be no idleness, no poverty, no physical misery, for the world's millions. With proper management there is more than to spare! The very misery of the world challenges us to this vision. *There is nothing inherently insufficient about the world's supply of matter and energy.*

In those four magic words: space, time, matter and energy, are contained much of man's knowledge, whether it be of evolutionary conceptions, economic theory, mechanical principles, or of chemical analysis—with all their enormous consequence for the practical arts and sciences—agriculture, industry, medicine, commerce, the military arts and so on.

There are people who stop here. They say that what has been outlined above pretty much embraces all there is to life; whatever else remains is a kind of residue, a sort of effect of such causes, and mainly accidental or fortuitous in character. Should you encounter sorrow and loss—that's too bad, but still incidental—your main attribute is that you are a possibly useful nucleus in a total assemblage of space, time, matter and energy.

Whatever sublime elements there are in such a universal conception, and there are such elements in it, it is not dificult to see how it becomes the ready servant of the unscrupulous, the embittered, and the rapacious in our midst. There is inherent in such a scheme no restraint of personal power, no law of conduct, no tradition of courtesy, no obstacle to barbarism. Yet its universality is apparent, and its very sublimity tempts men to all the deviltry which we now know, if we were ignorant before, unrestrained human activity is capable of.

Here then in outline is materialistic philosophy. Whatever particular name or form it may take, its bases are the same, and its power is the same, for *it is rooted in important universals.*

Taken alone it is a false philosophy, and if our mathematics and science teaching encourage such a world outlook, then then such teaching is for slavery and not for freedom. It is natural enough to see how such a philosophy can be taught often unwittingly by the teachers of mathematics and science because it is based upon the very elements with which mathematics and science deal. Somehow the student must see, and see clearly, much more, or else our education is education for further catastrophe.

Space, time, matter and energy do not of themselves form an adequate description of the basic elements in the world scene.

There is not time to discuss the phenomenon of life emergence, nor the gradations of life forms. Rather we shall assert that of all living phenomena, the human is the most significant. Hear the modern biologist:

> It is not mere anthropocentrism to assert that man is the highest product of evolution to date: it is a statement of simple biological fact. . . . Also . . . (1) the human species is now the sole repository of any possible progress for life . . .
>
> . . . it is a biological impossibility for any other line of life to progress into a new dominant type—not the ant, nor the rat, nor the dog or ape. (2) With the evolution of man, the character of progress becomes altered. With human consciousness, values and ideals appeared on earth for the first time. . . . The quest for truth and knowledge, virtue and goodness, beauty and aesthetic expression, and its satisfaction through the channels of science and philosophy, mysticism and morality, literature and the arts, becomes one of the modes or avenues of evolutionary progress.[1]

The self has certain obvious characteristics, which are not describable in space, time, matter and energy terms. Humility, envy, cooperativeness, greed or courage are not measurable in grams, centimetres and seconds.

The problem of self-hood is the central problem of each child born in each genera-

[1] Huxley, J. S., *Hibbert Journal*, April 1943.

tion. The problem is the recognition of *self-centredness* as the focal point of human misery, and the necessity for self-displacement before genuine self-growth is possible. Someone has said that the six greatest words in the world are: "Know yourself" (Socrates), "Control Yourself" (Cicero), and "Give (Lose) Yourself" (Jesus).

Now the plain fact is that the multiplication of material goods and comforts does not provide the means for self-growth; often it serves to act in the opposite direction.

During these last terrible generations it is as though the glittering triumphs of materialistic thought had bewitched us with their magic spell, luring us to our doom by their false promises, which in effect were appeals to sheer idolatry. We are awaking out of this spiritual stupor, I believe, to realize that there is more to life than material goods and comforts.

But it is not enough to awake out of stupor. We should realize that we have been misled as to the bases and objectives of living. We shall always be prey to soft philosophies unless we get grounded in the great spiritual verities of our highest civilization.

The lesson which we must learn is that our universe is much deeper in its nature and meaning than space, time, matter and energy. It is morally constituted, but more, at its heart are the whatsoevers of sublime goodness, wisdom and beauty. Those who ignore these majestic universals do so at their own life-peril. Children must come to see that true greatness is to be found in terms of absolute integrity of conduct, in self-control of appetites, in loving, skilful, self-abnegating service to their fellows.

Moreover, they must see that any other life-groundwork will lead to self-frustration, however fine looking are its pretensions.

These are hard sayings, but it is a bitterly disillusioned world we look out upon. We are being taught again that material aggrandizement is the sure way to spiritual anarchy.

We have sought material goods first and foremost, neighbor if and when expedient, and God—well, long since such a superstition has been dispelled—how?—in terms of space, time, matter and energy! And so the blind have led the blind. The first command (and a command is not an "if you please" or a "would you mind," but a stark imperative) is: Love God with everything you have; the second is: Love neighbor as self. Then, and then only, will material goods and the human relationships involved in them get put to rights.

Children and grownups alike must come to see that *what is right* is fundamental to a sane society. In the old phrase it is "righteousness alone that exalteth a nation." What is right means what is right in your home conduct; in your conduct as teacher; in your conduct as colleague. This is far more important than our prestige, our pose (which even a small child can see through!), our easily offended feelings, and so on!

Our quest for material comfort and security is essentially a soft thing—until our pseudo-rights to possession are challenged. Such a quest leads to no educational philosophy worth having, for it is basically acquisitive rather than co-operative.

But putting *what is right* first is the harder part, for it means a battle; we need fighting quality; life is a warfare perpetual; there is continuous need for alertness, courage, faith, confidence, and tolerance. Indeed the whole enterprise of exalting righteousness, that is, of putting things to rights and keeping things right requires battle fitness.

It is just here that educational practice has so often failed us. The hard sayings, the need of inner quality, the vision of life as a battlefield, have been displaced through a fear of being thought narrow or strict.

If a space-time-matter-energy concep-

tion of life is not enough, and if to it we must add the conception of what is right (good, true, beautiful), then surely such an altered outlook will affect our teaching of every subject.

The teaching of mathematics will have its contribution to make to the well-being of the child in the sense above described.

What is most important, however, is that the teacher (1) be persuaded that the outlook suggested is necessary for a sane society, and (2) practice with might and main to make that outlook a reality in his or her own life. It is only out of (1) and (2) that can come the insights by which our teaching is filled with the direction and the bite and the enthusiasm which it must have to get genuine results.

I. *The major aim of mathematics teaching, at least for the many, is to achieve some degree of competence in quantitative thinking.* Its concern is with those universals about which we spoke at the outset, namely, space, time, matter and energy. Nothing that I have said in the foregoing is to *tone down the basic importance of quantitative thought.* All we have asked for is that the pupil learn from the teacher of mathematics and science *that quantitative thought alone can never furnish an adequate basis for living.*

It is necessary but by no means sufficient. As long as teachers and pupil are seeing this, the lessons of quantitative thought can be learned with thoroughness; and they are good lessons.

It will be seen, moreover, that here is an approach at genuine integration of learning in school, where the teachers in each field make plain the limitations of the segment of life with which they deal. This will increase the chance for the pupil to see that each segment has bearing to and counterpoise for the others. History has suffered from one-sided emphasis, so has mathematics; so has literature. Literature needs science; conversely intelligence needs beauty and sorrow and faith and love for its completion.

II. The competence in quantitative thinking noted in I requires of the teacher a sound foundation in mathematical and scientific principles. We need more well-qualified teachers who see the fascinating opportunities which quantitative thinking can present.

Without such teachers, the real fruits of teaching mathematics and science in the school can neither be grown or garnered.

III. Now for some observations about quantitative thinking, as I have called it.

(1). Numerical thinking is still a challenge to the schools. Too few people feel at home with number situations. It seems incredible that people could prefer the saving account to victory bonds at twice the rate of interest—yet it is so, although the reasons may not be as simple as $2\times1\frac{1}{2}=3$.

(2) Geometry teaching must become still more real spatially. Probably our geometry is still too immaterial—we need material mathematics mixed up with spatial mathematics. Thus the mathematics of tin-cutting is both spatial and material.

(3) We have to make algebraic reasoning more real. The unreality for most pupils of the laws of indices, the occurrence of functions in the science laboratory, the frequency distribution, the problem of language translation and expression; these suggest that we can teach the marvelous symbolism of Algebra more effectively.

(4) The mathematics of space and time; the measurement of change; the problems of growth and decay; these areas will require imaginative exploration, and again would serve to bring mathematics and science teaching more closely together.

(5) The lessons which can be learned in dealing with quantitative data have bearing on clear, honest thought in general. Our mental health is dependent upon factors which so often show up in handling data: bias, prejudice, desire, anxiety, etc., and upon our knowledge of

word meanings, of the role of assumptions or convictions, etc.

At the outset I said something about the deeper, slower processes of life. The process of self-growth is such a process and is the true concern of education. Human nature can and does change, and it is our task to ensure that the change is made in wholesome and integrative directions.

Quantitative thought is one of the major aspects of the educational process. The part it plays in self-growth can be considerable, for its concepts and their application are indispensable to the development of a sane society. But such thought must never be divorced from the non-quantitative aspects of life which concern what is right and what is good.

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Some Forgotten Areas of Instruction in Mathematics

By Arthur M. Gowan
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In many areas of instructions it often happens that in our desire to cover the prescribed material, we assume that some of the fundamental skills of the area of instruction will be developed by themselves more or less automatically. The instructor evidently feels that if he succeeds in putting across the more difficult concepts and techniques that these fundamental skills, because they are a part of every problem, will be taken care of without any specific attention. It appears that such is the case in the teaching of elementary and secondary mathematics.

The statements and suggestions in this article are the outgrowth of two years of experience as supervisor of mathematics in the Naval Training School (Electrical) at Iowa State College. This school was established for the purpose of training Navy men for the rating of electrician's mate, and about 7,000 students from all states of the union were trained. It was necessary that the mathematics taught be of a functional type. Unless a certain technique was applicable to a practical situation, it was discarded. It was very definitely shop mathematics and was not taught as a background for further mathematical study. It was very apparent that the traditional mathematics teaching seemed to be neglecting certain fundamental skills that are essential when it becomes necessary to put mathematics to use in a practical situation.

Mathematics is considered a tool subject. However, it is one thing to know what tool to use, and another to know the technique for using the tool, and still another to be able to judge the results. In most mathematics teaching, the knowledge of these tools and their techniques for use have been given considerable emphasis, but too little emphasis has been placed on judging results. It is the purpose of this article to point out certain weaknesses and to attempt to suggest ways in which they may be corrected.

It could be said that in all calculations, we are interested in arriving at a reasonable, sufficiently accurate, and correct solution. Of the 7,000 students previously mentioned, very few had had any training in recognizing a reasonable answer. To most of them, degree of accuracy was a new term, and all too few had any regard for the correctness of a solution.

These principles raised for consideration apparently have received all too little attention in the teaching of mathematics. They are skills that must be emphasized and reemphasized through all the years a student is studying the basic fundamentals of mathematics. It cannot be given in a section in an eighth grade mathematics text and then assumed to be taken care of. Students should know what accuracy is and then accuracy should be demanded.

The first skill for consideration that seems to have been overlooked in mathematics teaching in general, judging by the experience with these 7,000 students, is that of recognizing when an answer is reasonable. This should be given consideration when striving for a correct result. Apparently very few students had ever been given any training in recognizing the reasonableness of an answer. It might be that such a condition has developed from too general use of answer books in elementary and secondary mathematics. Because of these answer books the student works for agreement with an already calculated result. He, therefore, has no need to give consideration to the

reasonableness of his result. It is true that students may be able to work more problems and perhaps get more practice in a certain method by having answers available, but this skill in method is being acquired at the expense of as equally an important a skill—that of being able to recognize a reasonable result.

But it seems that answer books are held to with great pertinacity. Any student should recognize that when a number is divided by an integral divisor larger than one, that the quotient, to be reasonble, must be less than the dividend and, too, when a number is divided by a divisor less than one, the quotient to be reasonable, must be larger than the dividend. He should recognize that if $3x=12$ that x must be ~~larger~~ smaller than 12. The above only provides for a reasonable answer, but certainly that must be given first consideration, when viewing a result for accuracy. The skill would be better developed if text book authors would omit many of the answers and if teachers would have answers viewed from the point of being reasonable. Many errors due to improper location of the decimal point could be eliminated if the student would look at his answer in the light of its being reasonable. It is evident that the development of such a skill will require emphasis over a long period of time and in a great variety of situations. It is a skill so much needed in practical situations that it should not be overlooked in our educational process.

After we have a reasonable answer, the next step for consideration is to determine its degree of accuracy. This is another area that seems to have had practically no consideration in the years of acquiring mathematical skills. In connection with degree of accuracy it is necessary that the student know what is meant by significant figures. Fewer students were familiar with this term than with degree of accuracy. Students should recognize that an answer of 203. has the same degree of accuracy as 0.0203. Both show accuracy to three significant figures. If an answer corrected to three significant figures is desired, the result should be carried one place further and rounded properly. In most cases if a student were asked to divide 8,036,742 by 3, he would undoubtedly divide until he got 2,678,914; but if this same student were asked to divide 0.025 by 3, he would probably get 0.008 and stop—not recognizing that if the first answer should be carried as far as it was, the latter should be 0.008333333 in order to have the same degree of accuracy.

If the situation requires accuracy with a 1% allowable error, the student should recognize what number of significant figures will give this degree of accuracy. It should also be stressed that when dealing with a group of data, that the degree of accuracy of the result can be no greater than the degree of accuracy of any single item of the data. Consideration to the above will be repaid by preventing much wasted effort in calculations. Any result carried past the point of accuracy required is lost motion, and sometimes gives an erroneous opinion as to the degree of accuracy of the result. It is true that a student cannot be expected to learn the degree of accuracy that accompanies all practical situations, but he should be acquainted with the term and the technique for determining what degree of accuracy a certain result has. He should know that there is a definite relationship between the number of significant figures in his result and the degree of accuracy of that result.

The skill can be acquired by requiring problems in which the student must make a decision as to how far an answer should be carried, and, of course, that means the answer book, as now used, is out. One particularly erroneous concept that some students had was that degree of accuracy was definitely related to the number of decimal places in an answer. Consideration of the above statements will show how misleading such a concept is.

We have discussed the need for acquiring the skill for recognizing a reasonable answer and then the degree of accuracy of an answer. Now what about the demand for accuracy? All through this discussion we have stressed the acquiring of these skills for application in practical situations. So let us continue in the same vein. When a man is on the job, what is the attitude of the employer toward a correct method with incorrect results? Such an obvious answer needs no space here. In the 7,000 students previously referred to there seemed to be much disregard for a correct answer. So frequently the question was raised: "Do we get credit for method?", when a result was very much in error due to careless calculation. The disregard, however, began to lessen when a student calculated the flow of current in a circuit and then, due to careless calculation, burned out the meter when it was placed in the circuit. A former student writing of his experiences on board a submarine remarked, "When the whole crew's safety depends on the accuracy of your calculation you begin to wish that accuracy had been given the place it should have had during your years in mathematics." Recently a large machine was to be installed in an industrial plant. An employee had been instructed to install the necessary fixtures for fastening the machine securely to its base. Finally, with great effort, the machine was raised to be placed in position, but due to some miscalculations the fixtures did not fit. An employee was discharged. The correct method may have been used but accuracy was demanded. It seems very apparent that the aforementioned skills are very necessary to cope with the situations that arise in the solution of so many practical problems.

Using the results of the teaching of mathematics to the 7,000 students as a basis for judgment, it seems quite apparent that these skills have not been sufficiently developed. They are skills that, while basically the same in all problems, require many varied situations for development. They are skills, the development of which should be started early. These skills should be presented concomitantly with the basic skills in mathematics. From the first instruction in mathematics, the student should be instilled with the idea that a correct answer is the only acceptable answer. From the beginning students should be required to investigate and weigh the result from the point of reasonableness. Degree of accuracy could not be introduced as early but certainly should be undertaken by the middle of the junior high school mathematics.

The following suggestions should be helpful in developing these skills:

1. Reduce greatly or eliminate entirely answers that accompany most texts.
2. When problems are completed, have the answer discussed from standpoint of reasonableness.
3. A correct result to a specified degree of accuracy should be required at all times.

To accomplish these will require no drastic change in any text book or course of study. It requires only that certain skills, much needed in practical situations, be emphasized in textbook situations.

Demonstration of Conic Sections and Skew Curves with String Models

By H. von Baravalle
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For the demonstration of conic sections in high schools and colleges, we generally use the wooden model of a cone with detachable parts, showing the four kinds of plane sections, a circle, an ellipse, a parabola, and a hyperbola. The wooden model is naturally limited to fixed positions of the intersecting plane and the conic sections appear as four separate facts. As one of the most stimulating parts of a study of conic sections is the realization of how each one of the four curves changes with the position of the intersecting plane and how one kind of conic section can turn into another, a more flexible medium of demonstration seems desirable. It can be found through applying two changes to the usual demonstrations. The first one is to replace the actual cutting of a solid wooden model by projecting a cut on a larger cone, thus achieving an easy flexibility, as either the projector or the model can be moved during the demonstration. The second change is to replace a solid surface by one formed by strings, which not only makes a larger model considerably lighter, but also allows the conic sections to be seen all around the cone. The alternative to string models, models with transparent surfaces, produce too many disturbing light reflections. Figure 1 shows the string model of a cone and Figure 2 its application in the demonstration of ellipses. The system of parallel ellipses in Figure 2 is produced by parallel light planes which are obtained from an ordinary slide projector with slides showing transparent lines on black background. Instead of glass slides also rectangular pieces of stronger drawing paper can be used into which the lines have been cut. Moving the projector slightly to the side makes the ellipses in Figure 2 go up or down the cone and

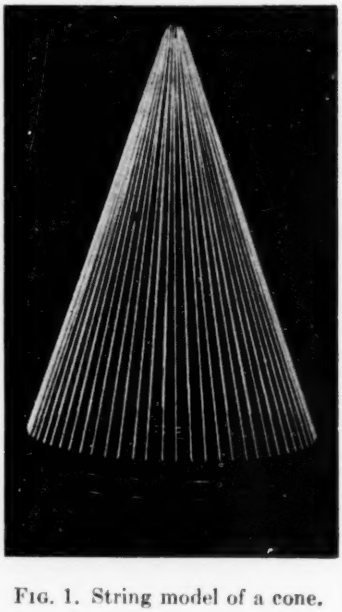

Fig. 1. String model of a cone.

Fig. 2. String model of a cone during the demonstration of ellipses.

each ellipse widens or contracts during the motion. If one increases the angle of the planes the form of all the ellipses change until they turn into parabolas and finally hyperbolas.

A comparison between conic and cylindric sections can be carried out with the additional aid of a cylindric string model (Fig. 3). On the cylinder all plane sections

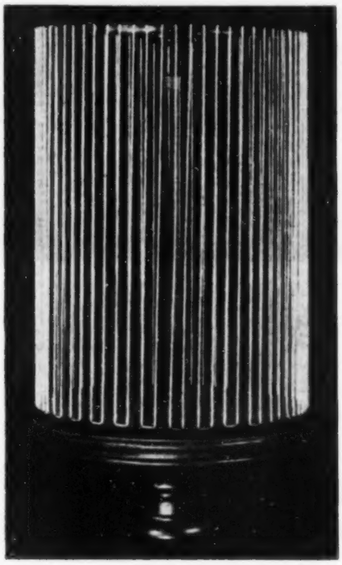

FIG. 3. String model of a cylinder.

are ellipses (including circles and two parallel lines in special cases) no parabolas, no hyperbolas. In their motions up and down the cylinder the ellipses do not change their form nor size. A slide with concurrent straight lines projected on the cylinder produces ellipses of varied eccentricity intersecting in two points.

Through its simpler form, the cylinder is particularly suited for proceeding, with it to the demonstration of skew curves of higher than the second order. By replacing the slides showing straight lines by slides with circles, ellipses, parabolas, etc., or by curves of higher order, one can demonstrate intersection of the cylinder with light cones of various directrices. Figure 4, for instance, is the photo of a skew curve produced by the intersection of a parabolic light cone with the

FIG. 4. Skew curve on a cylinder.

circular cylinder of a string model.

The variety of demonstrations can be still further enlarged by the use of further models and of flexible surfaces. Figure 5 shows a string model of a one sheet hyperboloid. The following figures, Figures 6 and 8 show a flexible model which changes its form from a cylinder to a cone via various hyperboloids through a rotation of the top circle. A mechanical device is necessary to keep (during this operation) the length of the strings constant. It has been achieved by a corresponding

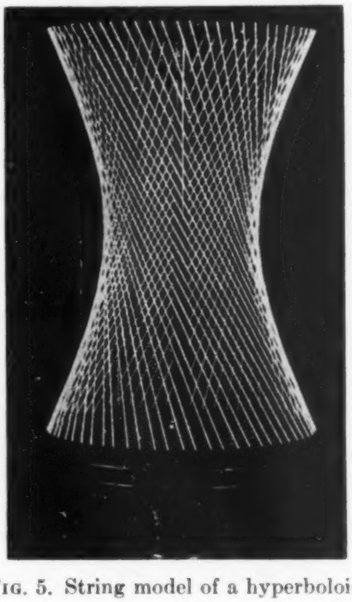

FIG. 5. String model of a hyperboloid.

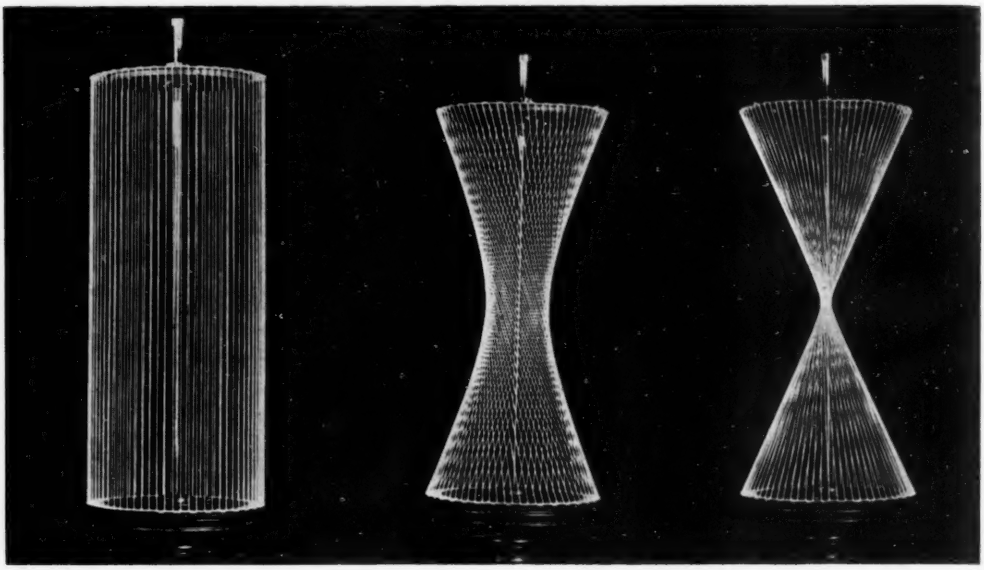

FIG. 6
A cylinder.

FIG. 7
Flexible string model showing a hyperboloid.

FIG. 8
A double cone.

lowering of the top circle during the operation (constructed by C. Schleicher; Threefold Workshop, New York, where the models are now reproduced for the use in schools and colleges).*

* Threefold Workshop, 318 West 56th Street, New York City.

In classes of solid geometry or in courses of descriptive geometry, string models can also be used to demonstrate sections of regular solids. For this end, models made of wires following the edges of the solids are equipped with parallel strings all along their faces. A photo of the

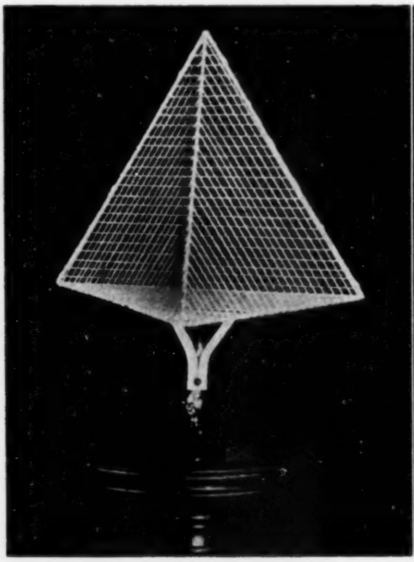

FIG. 9. String model of a tetrahedron.

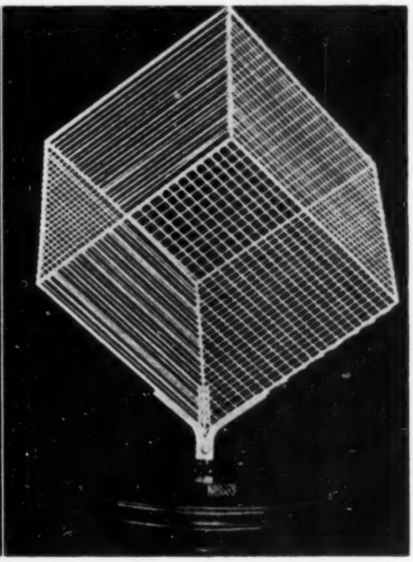

FIG. 10. String model of a cube.

string model of a tetrahedron is reproduced in Figure 9 and one of a cube in Figure 10. Holding a string model into one or several light-planes shows their plane sections. Among those are for the tetrahedron: equilateral triangles of various sizes, rectangles and squares; and for the cube: equilateral triangles, rectangles, squares, and regular hexagons. A circle projected on one of these models shows the skew curve of the tetrahedron or cube with a light cone. By using finally a slide showing a system of curves for projection, either on a plane or on a curved surface, a system of skew curves is produced which in turn can be shown in motion if one moves (during the demonstration) either the projector or the strong model or both.

Senior High School Mathematics Differentiated According to What Needs?*

By MARGARET MCALPINE
McAlester High School, McAlester, Okla.

WE HAVE been hearing a great deal lately about mathematics in the senior high school differentiated according to needs. What are these needs of senior high school pupils? How can they be determined?

Needs of people are of two kinds; present needs and needs of the future. The recent proposal of the Commission on Post War Plans of the National Council of Teachers of Mathematics on post war mathematics in the senior high school emphasizes needs of people in industry, business, professions and the armed forces for mathematics. These are needs by people out of school for mathematics; needs in mathematics of our high school pupils in the future.

Present needs of people are usually determined by their experiences. A carpenter has need for a hammer, for he uses it to drive a nail. He is having experiences now with driving a nail; hence his need for the hammer. Why not search, then, in senior high school pupils current experiences for their needs in mathematics? Why must we build our courses for senior high students on their future needs alone? Why not include some of their present ones? Why not include some needs determined by their experiences now?

What kind of experiences do senior high school students have that would give us a key to their needs in mathematics? We might divide their experiences into two categories—in-school experiences and out-of-school experiences.

Those out-of-school experiences might include work after school, church, social clubs, or experiences in the home. The in-school experiences are connected with the students academic program. For example, the experiences of a freshman in high school might include English, General Science, Citizenship, Physical Education, and possibly some form of music.

The latter experiences, the in-school ones, can possibly give us more help in finding present needs of high school students in mathematics. By studying the subject matter of courses other than mathematics we can determine some of their current needs in mathematics from their in-school experiences.

Let's examine the proposal of basing courses in mathematics for all senior high students on their future needs. Let's look at its possibilities with freshmen and sophomores in high school. How meaningful are these future needs to ninth and tenth grade people? How much use do freshmen have for a knowledge of income tax computation or budgeting? Why should we teach insurance, investment, budgeting, and taxes to ninth and tenth grade pupils when they have little to insure, nothing to be taxed, little to budget and nothing to invest? Would it not be better to teach some applications which they can use now? Would it not be better to build courses in mathematics on senior high school students needs now, determined by their experiences now? If the students do not see the need or have no interest in the applications then it is little better than cramming theoretical algebra down their throats as we have done in the past!

The reply to this might be that we are girding them for future jobs. For freshmen and sophomores this is doubtful. Juniors

* Miss McAlpine says that this paper resulted from a stimulus obtained from reading recent issues of THE MATHEMATICS TEACHER, but that "the fire which really set off the verbal steam was Benjamin Braverman's article in the October 1945 issue—Editor.

and seniors in high school would be more interested in the proposal of the National Council for they are nearer to those experiences where the needs arise. For seniors some of the experiences are met within the next few months. For freshmen and sophomores, they are four years in the future.

What then, in place of the proposal of the National Council for social and business mathematics for senior high freshmen and sophomores?

It is necessary to have a mathematics used by all students and the need of which is seen by all students. Let's construct courses to help pupils grasp those ideas needed in other classes for complete understanding of those subjects.

It is just as easy to learn to find averages in reading speeds in English as it is to find an average annual egg laying capacity of a bunch of make-believe hens! It is just as easy to measure objects used in General Science and weigh materials used in Home Economics as it is to measure some fixed lines drawn in the text book.

"Mathematics becomes a tool subject rather than mathematics for itself," some cry. What more sound idea could we request? Is not English a tool subject, science a tool, history a tool? Then why not mathematics?

Yes, mathematics courses for senior high students should be differentiated according to needs, but those needs should be of importance to those students now. They have needs of the future, true, but let's not forget those current needs determined by their present experiences.

We can fulfill *both* needs in senior high school mathematics courses—needs now and needs of the future.

The Teaching of Graphs

By Sister Noel Marie, C.S.J.

College of St. Rose, Albany, New York

One question on the January, 1946, question paper in Intermediate Algebra, in New York State, brings forceably to teachers' minds the double task lying before them when teaching graphs. The question, in part, is:

> Each of the following statements is sometimes true and sometimes false, depending on the value of a, b, and c. In *each* case assign to a, b, and c numerical values that will make the statement *true* and also numerical values that will make the statement *false*.
>
> (1) The graph of the parabola $y=ax^2+bx+c$ passes through the origin.
>
> (2) The graph of the equation $ax+by=c$ is parallel to the X-axis.
>
> (4) The roots of the equation $ax^2+bx-c=0$ are equal.

Let us consider, first, the question concerning the graphing of the line, $ax+by=c$. A teacher, ordinarily, instructs her class to set up a table of values that satisfy an equation of this form, locate the points corresponding to the set of values and then draw a smooth curve through these points. Obviously, this is only groundwork—and, in many instances, mechanical work. There are many opportunities for discussion of the graph.

This would involve the fact that:

1. a first-degree equation in two variables represents a straight line.
2. when $c=0$, the line passes through the origin.
3. if the line were written in the form, $y=mx+b$, m represents the slope and b the y-intercept. A further discussion of the line in this form enables the pupil to investigate the effect on the graph when m remains constant and b varies, and *vice versa*. Also the fact that lines which have the same slopes are parallel.
4. by substituting $y=0$, the x-intercept may be determined and by substituting $x=0$, the y-intercept may be found. The use of these two values, in most cases, yields a more satisfactory method of drawing the graph of a straight line.
5. a line of the form $x=a$ is parallel to the Y-axis and a line of the form $y=b$ is parallel to the X-axis. A recognition of this fact would, perhaps, be sufficiently advanced for a high school pupil's study.

There are many phases in the graphing of the quadratic $y=ax^2+bx+c$, the study of which enriches the concept of graphs. They may include the cases where:

1. the curve passes through the origin.
2. the curve is tangent to the X-axis. Variations in the study of this cases should include the determination of coefficients that will make the roots equal.
3. the nature of the roots, real or imaginary, equal or unequal, rational or irrational, is determined from a study of the graph.
4. the effect of the change of coefficients on the shape of the graph.
5. the fact that $f(x)$ and $f(-x)$, $f(x)$ and $2f(x)$ have the same *real zeros*.

Frequently, it is sufficient for the teacher to point out to the class the implications in a graph or the comparisons between graphs to make their students conscious of the richness of content found in the study of this unit. Dependence, variation and continuity have a new meaning, and a *pictured* meaning, which cannot help but be more vividly portrayed.

No teacher is justified, then, in feeling that she has "taught" graphs when her class is able to execute only the construction of them.

EDITORIALS

Kate Bell Retires

Miss Kate Bell head of the department of mathematics at the Lewis and Clark High School in Spokane, Washington, has retired from active service.

Miss Bell, who received all her elementary education in Emporia, Kansas, graduated from the University of Chicago with a B.S. Degree. She also did graduate work at the Universities of Minnesota, Michigan, Chicago, Columbia, and Washington.

As one of her special interests, Miss Bell lists traveling. "I have seen quite a bit of the United States," she said, "but there are two things in the Northwest I want to do that I haven't been able to do as yet. One is to take a trip up Lake Chelan and the other is to see Hell's Canyon on the Snake River." Miss Bell also said that she would like to take a trip down through Bryce Canyon and Zion National Park.

Before beginning her teaching career, Miss Bell was the head of the Mathematics department at the high school at Lead, South Dakota. During recent years, she taught summer school in the School of Education at the University of Washington.

In 1936, Miss Bell exchanged positions for one year with Miss Hope Chipman, a teacher in the University High School at the University of Michigan.

She was a member of the Board of Directors of the National Council of Teachers of Mathematics for six years from 1938 to 1944. She is a member of Phi Lambda Theta, an educational honorary society for women.

Miss Bell stated that she had prepared to teach mathematics and had never taught in any other high school field.

In reference to her life work, Miss Bell said, "I enjoyed my work at Lewis and Clark immensely and shall regret leaving. Do not believe any rumors you hear about what I am going to do because I have not the slightest idea about it myself."

"June 7 marks for Miss Bell the close of a long period of teaching service in this school and community," said Principal A. L. Parker. "Her friends among the graduates of Lewis and Clark are legion. Her friends, graduates, faculty members, and the student body will miss her active participation in this school's affairs. What Lewis and Clark owes Miss Bell can never be repaid. Her friendly guidance, her words of council and her influence nationally in the field of mathematics will always be a part of the Lewis and Clark tradition."—W.D.R.

The Passing of Two Great Educators

Two great American educators passed away during the summer and their loss will be keenly felt by education generally and by mathematics teachers in particular for both of these men were real friends of mathematics. Both knew something of the nature of mathematics and its importance in the education of American citizens and they did not hesitate to give a great deal of time and thought to place mathematics in the right place in the minds of the public. The first of these, Dr. William Chandler Bagley, Professor Emeritus at Teachers College, Columbia University, and editor since 1939 of *School and Society*, a weekly educational magazine, died on July 1 at his home, 200 West Fifty-eighth Street, New York City. He was 72 years old.

A member of the "Essentialist" school of educational philosophy, Dr. Bagley was a frequent critic of progressive education. "Similar theories," he once warned, "caused a softening of the fiber of Greek

education 2,500 years ago and doubtless played a part in the decadence of Greek civilization."

Dr. Bagley was a firm advocate of sound methods in education, with the stress on discipline and on mastery of simple fundamentals first.

Born in Detroit on March 15, 1874, Dr. Bagley was a son of William Chase Bagley and the former Ruth Walker. He attended Michigan State College, obtaining a B.S. degree there in 1895. In 1898, he received a Master of Science degree at the University of Wisconsin. Two years later the Ph.D. degree was conferred on him at Cornell.

When the "Essentialist" movement was founded in 1938, Dr. Bagley and his associates laid at the door of progressive education charges that the average child in American elementary schools did not measure up to his European counterpart, that American high school students could not read effectively and were unproficient in simple arithmetic and English grammar. The expansion of the education program "has not been paralleled by a significant decrease in the ratios of serious crime," the "Essential manifesto" also stated, therein returning to a charge long a favorite of Dr. Bagley's—that some modern methods and the elimination of a firm masculine jurisdiction in the school and the home were major causes of crime.

He leaves a widow, the former Florence MacLean Winger of Lincoln, Neb.; a daughter, Mrs. William B. Cobb, and a son, William C. Bagley, Jr.

The second of these men, Dr. Charles Hubbard Judd, emeritus Charles F. Grey distinguished-service professor, the University of Chicago, died at his home in Santa Barbara (Calif.), July 18, at the age of seventy-three years. Dr. Judd who was widely known as an educator, author, and psychologist, had served as instructor in philosophy (1896–98), Wesleyan University; professor of experimental psychology (1898–1901), New York University; professor of psychology and pedagogy (1901–02), University of Cincinnati; instructor in psychology (1902–04), assistant professor (1904–07) and professor and director of the psychological laboratory (1907–09), Yale University; head professor of education (1909–38), the University of Chicago; director of education (1938–40), National Youth Administration; and consultant for the public schools of Santa Barbara (1945–46). His work in the field of psychology, both through his writings and through his lectures, exerted a profound influence on American education. His last address, "Teaching the Evolution of Civilization," was delivered at the biennial convocation of Kappa Delta Pi in Milwaukee, March 12.

Dr. Judd's well known book *The Psychology of Secondary School Subjects* contains a great deal of valuable material on mathematical education which every teacher of secondary school mathematics should read.—W.D.R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
Midwood High School, Brooklyn 10, New York

The American Mathematical Monthly
March 1946, Vol. 53, No. 3.

1. Kline, J. R., "Rehabilitation of Graduate Work," pp. 121–131.
2. Bancroft, T. A., and Winger, R. M., "Mathematics at the American Universities in Europe," pp. 131–135.
3. Coxeter, H. S. M., "Quaternions and Reflections," pp. 136–146.
4. Discussions and Notes, pp. 147–150. (a) Mancill, J. D., and Thomas, Betty, "On the Equation of Joukowski's Aerofoils."
5. Clubs and Allied Activities, pp. 150–153.
6. Recent Publications, pp. 154–156.
7. Problems and Solutions, pp. 156–164.
8. News and Notices, p. 165.
9. General Information, pp. 166–172. (a) Schorling, Raleigh, "The A.A.A.S. Cooperative Committee on Science Teaching." (b) "The Report of the Harvard Committee."
10. The Mathematical Association of America, pp. 172–180.

April 1946, Vol. 53, No. 4.

1. Blumberg, Henry, "On the Change of Form," pp. 181–192.
2. Cochran, W. G., "Graduate Training in Statistics," pp. 193–199.
3. Goormaghtigh, R., "Pairs of Triangles Inscribed in a Circle," pp. 200–204.
4. Discussions and Notes. (a) Gloden, A., "Two Theorems on Multi-degree Equalities," pp. 205–206. (b) Karmelkar, S. M., "Construction of the in-Feuerbach Point," pp. 206–207. (c) Frame, J. S., "Note on Logarithms and Compound Interest," pp. 216–217.
5. Club and Allied Activities, pp. 217–218.
6. Problems and Solutions, pp. 219–228.
7. News and Notices, pp. 228–232.
8. General Information, pp. 232–236. (a) "Release of Faculty Personnel from the Services." (b) "Mathematics Required for Graduation in Kansas."
9. Mathematical Association of America, pp. 236–240.

School Science and Mathematics
April 1946, Vol. 46, No. 4.

1. Goodrich, Merton T., "The Use of Mathematics in the Beautification of School and Home Grounds," pp. 313–317.
2. Ives, Ronald L., "Computations in Precision Riveting," pp. 323–328.
3. Carnahan, Walter H., "History of Arithmetic" (continued), pp. 329–334.
4. Graeser, R. F., "Some Mathematics of the Honey Comb," pp. 339–343.
5. Jerbert, A. R., "The Inverse Distributive Law," pp. 351–358.
6. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 372–375. (116) Transposition. (117) Meaning, Thinking and Drill. (118) Interference Bonds.

May 1946, Vol. 46, No. 5.

1. Schaaf, William L., "Arithmetic Taught as a Basis for Later Mathematics," pp. 413–423.
2. Betz, William, "Central Issues in the Program of Mathematics for a World at Peace," pp. 446–447.
3. Junior High School Mathematics
   (a) Spangler, Mamie, "Why Is the Ability to Recognize Relationships in Reading of Problems so Vital and What Methods May be Used to Develop this Ability?" pp. 448–452.
   (b) Butler, Charles H., "References on Improving Reading Competence in Mathematics," pp. 452–459.
   (c) Murray, Walter I., "How Can the Reading of Context and Explanatory Material Be Improved so that the Student will Have a Clear Understanding Before Proceeding into Actual Problem Solving?" pp. 459–463.
   (d) Blakely, Bernice, "In What Ways Can the Mathematics Teacher Develop a Meaningful Mathematics Vocabulary and Not Destroy the Meanings those same Words May Have in Outside, Situations?" pp. 463–467.
4. Mahin, Albert R., "Do We Need Refresher Mathematics?" pp. 471–479.

Scripta Mathematica
September-December 1945, Vol. 11, Nos. 3–4.

1. Editorial Note, p. 207.
2. Reeve, William David, "David Eugene Smith," pp. 209–212.
3. Archibald, Raymond Clare, "Mathematical Table Makers—Portraits, Paintings, Busts, Monuments, Bio-Bibliographical Notes" (I), pp. 213–245.
4. Quotations, pp. 246, 263.
5. Simons, Lao G., "Among the Autograph Letters in the David Eugene Smith Collection," pp. 247–262.
6. Kasner, Edward, "The Recent Theory of the Horn Angle," pp. 263–267.
7. Karpinski, Louis C., "The Place of Trigonometry in the Development of Mathematical Ideas," pp. 268–272.
8. Curiosa, pp. 273–274.
9. Reeve, William David, "Mathematics in the Post-War Period," pp. 275–307.
10. Bell, E. T., "Possible Projects in the History of Mathematics," pp. 308–316.
11. Kraitchik, Maurice, "On Certain Rational Cuboids," pp. 317–326.
12. Frick, Bertha, "The First Portuguese Arithmetic," pp. 327–339.
13. Ginsburg, Jekuthiel, "Iterated Exponentials," pp. 340–353.
14. Keyser, Cassius Jackson, "The Significance of Death," pp. 354–356.
15. Book Reviews, pp. 357–359.
16. Recreational Mathematics, pp. 360–363.
    (a) Grossman, Howard D., "The Twelve Coin Problem."
    (b) Withington, Jr., Lothrop, "Another Solution of the 12-Coin Problem."
    (c) Shedd, Charles L., "Mathematical Arches."
17. Miscellaneous Notes, pp. 364–379.
    (a) "David Eugene Smith—A Sketch."
    (b) Smith, D. E., "The Wise Man of Sumatra."
    (c) "Extracts from Letters to Dr. Smith on His Seventy-Fifth Birthday."
18. Moorman, R. H., "A Lower Limit of the Roots of a Rational Integral Equation," pp. 379–380.

NEWS NOTES

Ralph Grafton Sanger, member of the mathematics faculty and dean of students in the division of physical sciences at the University of Chicago, has been appointed new head of the mathematics department at Kansas State, it was announced by President Milton S. Eisenhower.

Dean Sanger, whose appointment is effective September 1, will succeed Dr. W. T. Stratton, who will retire from administrative duties after nine years as department head. Dr. Stratton will continue as a full-time member of the department.

The Suffolk County (N. Y.) Mathematics Teachers Association which was inactive during World War II held a reorganization dinner meeting on May 7th at which the following officers were elected:

President—Miss Elizabeth Ormsby of Bayport, Vice-President—Miss Lillian Vogel of Huntington, Secretary-Treasurer—Miss Leona Hirzel of Sayville. Plans were made to have programs for the coming year.

The last meeting of The New York Association of Mathematics Teachers was held on May 24, 1946 at 8.15 P.M. in the Washington Irving High School. The speakers were:

1. Dr. Nathan Lazar—Is the analysis of the indirect proof valid? Every teacher of mathematics will be interested to learn that the traditional analysis of the indirect proof as presented in geometry textbooks is logically unsound. An alternative proof, using the Law of Inconsistency, will be recommended.
2. Mr. Morris Hertzig—The place of mathematics in the curriculum. Teachers must be aware of the fact that the lessons of the war, regarding mathematical education, are not being recognized by general educators.

A report on a recent publication of the State Education Department "Basic Issues in Secondary Education."

Ohio State University's department of mathematics announces several personnel changes as the fall quarter nears its opening October 1.

Retiring from the department staff after 30 year's service is Professor Grace M. Bareis, who first joined the Ohio State faculty as a graduate assistant in 1906.

New appointments to the staff include: Marshall Hall, formerly on the staff at Yale University, and Howard H. Alden, formerly of the University of Wyoming faculty. Hall, who comes to OSU with the rank of associate professor, served in the navy as a lieutenant commander. His field of specialization is algebra and group theory. Alden, an assistant professor at OSU, is a specialist in mathematical applications in engineering.

Two of the five resignations on the mathematics staff were submitted by professors who went into government service and the other three men transferred to other institutions. Resignations included J. L. Synge, professor and chairman, who went to Carnegie Institute of Technology as chairman of the department of applied mathematics; Vincent F. Cowling, instructor, who went to Lehigh University; Joshua Barlaz, instructor, to Rutgers University; and George E. Albert and Clarence R. Wylie, Jr., both of whom entered government service.

Professor Tibor Rado, professor of mathematics, has been named chairman of the department, and two instructors, Robert G. Helsel and Earl J. Mickle, have been advanced in rank to assistant professors.

MAJOR CONFERENCE

THEME: Toward Agreement on Purpose, Program and Design of Education for American Youth.

PLACE AND TIME: Teachers College, Columbia University, November 18th and 19th, 1946.

PROGRAM AND ORGANIZATION: Consideration of five special committee reports based upon President Conant's suggestions contained in *A Truce Among Educators*. The reports will be presented by the chairmen as follows:

Dean T. R. McConnel, University of Minnesota
Dr. C. Leslie Cushman, Public Schools, Philadelphia
Dr. Stephen M. Corey, University of Chicago
Dr. Karl Bigelow, Teachers College, Columbia University
Dr. Maurice E. Troyer, Syracuse University

A *Critique* of each report by an outstanding educator chosen because of his special competence to deal with the content of the report.

Discussion Group meetings open to conference-members, to discuss the contents of the reports with the five committees.

OUTCOMES: The revised reports will be laid before various institutions, organizations and associations responsible for the education of American youth as a proposed basis for more unified action.

PARTICIPANTS: Restricted to teachers and administrators in public and private secondary schools and higher institutions. Possibly your institution may wish to send a representative.

The Journal of General Education published quarterly by the University of Iowa, is intended to serve instructors and administrative officers in liberal arts colleges, professional schools, teachers colleges, junior colleges and the secondary schools. More specifically, it is designed to

provide an outlet for thoughtful discussions of the issues and experiments of general education. To assure representative and timely content, a board of editorial consultants assists the editor each quarter in selecting the papers to be included.

EARL JAMES MCGRATH, *Editor*

Board of Editorial Consultants: Byron S. Hollinshead, B. Lamar Johnson, Lennox Grey, W. H. Cowley, Raphael Demos, W. E. Wickenden, Doak S. Campbell, Lewis Mumford, William P. Tolley, T. R. McConnel, Robert J. Havighurst, John W. Harbeson.

Published October 1, January 1, April 1 and July 1. Subscription price: $2.00 per year. Editorial office: 108 Schaeffer Hall, Iowa City, Iowa. Checks should be made payable to *The Journal of General Education*, Business Office, University Hall, Iowa City, Iowa.

The Metropolitan New York Section of the Mathematical Association of America held its Fifth Annual Meeting at The Cooper Union, Cooper Square, New York 3, N. Y. on Saturday, May 4, 1946.

Program

Morning Session, 10:00 A.M., Room 205, Hewitt Building.
Chairman: Professor H. E. Wahlert, New York University.
"Address of Welcome," Dr. Edwin S. Burdell, Director, The Cooper Union.
"Cartesian Geometry from Fermat to Lacroix," Professor Carl B. Boyer, Brooklyn College.
"Geometry of Ship Waves," Professor J. J. Stoker, New York University.
"Evaluating a Syllabus in Experimental Geometry a Priori," Mr. Charles Salkind, Samuel J. Tilden High School.
Afternoon Session, 2:00 P.M., Room 205, Hewitt Building.
Chairman: Professor F. H. Miller, The Cooper Union.
Business Meeting and Election of Officers
"Coordinating High School and College Mathematics," Professor William D. Reeve, Teachers College, Columbia University.
"Infinity in Art," Professor Edward Kasner, Columbia University.

Officers of the Metropolitan New York Section, 1945–1946: Frederic H. Miller, The Cooper Union, Chairman; Howard E. Wahlert, New York University, Vice-Chairman; Carl B. Boyer, Brooklyn College, Secretary; Aaron Shapiro, Midwood High School, Treasurer.

The California Mathematics Council held its San Francisco Conference, on Tuesday, April 16, 1946.

Program

9:30—High School of Commerce, Room 34—Meeting in conjunction with California Association of Secondary School Principals.

Theme: *The Forward View in the Teaching of Mathematics*

Dr. Cornelius Siemens, University of California, "Pooling the Resources in the Teaching of Mathematics."
Dean L. B. Kinney, Stanford University, "The Challenge of the Report of the Commission on Post-War Planning of the National Council of Teachers of Mathematics."
Mr. Lawrence J. Hill, Vice-Principal, San Jose High School, "Guidance Implications of the National Report."

Luncheon and Afternoon Meetings will be held at Galileo High School.

12:15—Luncheon—Galileo High School.

Presentations:

Mrs. Cecile LaViolette Ehrhart, San Leandro Junior-Senior High School, Oakland Public Schools, "What Is Being Done in Guidance in the Mathematics Program?"
Mrs. Ruth G. Sumner, Oakland High School, "Summary of the Meeting of the National Council of Teachers of Mathematics at Cleveland."

1:15—Symposium—*What Can we Learn from the Armed Forces so as to Improve Methods of Teaching Mathematics in the Schools?*

Mr. Frank E. Johnston, Lowell High School, San Francisco.
Miss Laura Henry, Stanford University.

2:15—Election of regional officers of the California Mathematics Council for the Bay Area.

2:30—Discussion sections on *What Is Being Done to Improve the Mathematics Curriculum?*

a. In the Elementary School
   Chairman: Miss Margaret Hickey, LeConte School, San Francisco.
   Miss Georgia Davis, Supervisor, Modesto County Schools, "Forward Views of the Arithmetic Curriculum."
   Miss Irene Henderson, Supervising Principal, Gardner and Longfellow Schools, San Jose, "The New Arithmetic Curriculum in San Jose."
   Miss Eva Gildea, Supervisor of Student Teachers, San Francisco State College, "Concept Development in Arithmetic."

b. In the Junior High School
   Chairman: Mr. Lee Y. Dean, Principal, Franklin J. H. S., Vallejo.
   Mr. Charles Franseen, Principal, Roosevelt J. H. S., San Jose, "Improvements in Junior High Mathematics."
   Dr. Ralph Troge, Principal, Woodrow Wilson J. H. S., San Diego, "An Experiment in Ninth Grade Algebra."
   Mrs. Cecile LaViolette Ehrhart, San Leandro Junior-Senior High School, "Remedial Program in 7th Grade Mathematics."

c. In the Senior High School
   Chairman: Miss Edith Mossman, Richmond Union High School.
   Mr. Charles Fabing, Dorsey High School, Los Angeles, "The Problem of Non-College-Preparatory Mathematics."
   Mr. Donald Kauffman, Sequoia High School, Redwood City, "What We Should Do for College-Bound Students."

Dr. Charles J. Falk, Assistant to the Superintendent, San Diego City Schools, "Experimentation in Algebra."

Committees

Program: Mr. M. E. Mushlitz, Assistant Chief, Div. of Sec. Ed., State Dept. of Ed.; Miss Rachel Keniston, Program Vice-President, California Mathematics Council, Stockton High School.

Arrangements: Chairman: Miss Harriet Welch, Temporary Regional Director, California Mathematics Council, Curriculum Department, San Francisco Public Schools; Publicity: Miss Mary McBride, Lowell High School, Miss Aileen Hennessy, High School of Commerce, Mr. Caleb Cullen, Galileo High School; Reservations: Mrs. Carolyn Riedeman, Girls' High School; Registration: Miss Una McBean, Girls' High School, Mr. Harold Brillhart, High School of Commerce.

◆ NEW BOOKS RECEIVED ◆

1. Bell, E. T., *The Development of Mathematics.* McGraw-Hill Book Co., Inc., New York, 1945. 600 pp. $5.00.
2. Bell, Clifford, and Thomas, Tracy Y., *Essentials of Plane and Spherical Trigonometry* (Revised Edition). Henry Holt & Co., New York, 1946. 350 pp. $2.30 with tables, $2.00 without tables.
3. Braverman, Benjamin, *Gaining Skill in Arithmetic.* D. C. Heath & Co., Boston, 1945. 134 pp. $1.40.
4. Coolidge, J. L., *A History of the Conic Sections and Quadric Surfaces.* Oxford University Press, 1945. 214 pp.
5. Cornett, R. Orin, *Algebra, A Second Course.* McGraw-Hill Book Co., Inc., New York, 1946. 313 pp. $2.00.
6. Ewing, Claude H., and Hart, Walter W., *Essential Vocational Mathematics.* D. C. Heath and Company, Boston, 1945. 266 pp. $1.60.
7. Granville, William Anthony, Smith, Percy F., and Longley, William Raymond, *Elements of Calculus.* Ginn & Company, Boston, 1946. 549 pp. $3.75.
8. Keniston, Rachel P., and Tully, Jean, *Plane Geometry.* Ginn & Company, Boston, 1946. 392 pp.
9. Knight, F. B., Studebaker, J. W., and Tate, Gladys, *Mathematics and Life, Book 2.* Scott, Foresman & Co., Chicago, 1946. 512 pp.
10. Miller, Frederick H., *Calculus* (Second Edition). John Wiley & Sons, Inc., New York, 1939, 1946. 416 pp.
11. Murnaghan, Francis D., *Analytic Geometry.* Prentice-Hall, Inc., New York, 1946. 402 pp. $3.25.
12. Nowlan, Frederick S., *Analytic Geometry* (Third Edition). McGraw-Hill Book Co., Inc., New York, 1946. 355 pp. $2.75.
13. Schorling, Raleigh, and Clark, John R., *Mathematics in Life.* World Book Co., New York, 1946. 500 pp. $1.80.
14. Slade, Samuel, and Margolis, Louis, *Mathematics for Technical and Vocational Schools* (Third Edition). John Wiley & Sons, Inc., New York, 1922, 1936, 1946. 532 pp. $2.50.
15. Watts, Earle F., and Rule, John T., *Descriptive Geometry.* Prentice-Hall, Inc., New York, 1946. 301 pp. $3.00.
16. Wertheimer, Max, *Productive Thinking.* Harper & Brothers, Publishers, New York, 1945. 224 pp. $3.00.
17. Wolfe, John H., Mueller, William F., and Mullikin, Seibert D., *Industrial Algebra and Trigonometry, With Geometrical Applications* (First Edition). McGraw-Hill Book Co., Inc., New York, 1945. 389 pp. $2.20.
18. Wrightstone, J. Wayne, and Meister, Morris, *Looking Ahead in Education.* Ginn & Co., Boston, 1945. 151 pp. $1.50.